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Geometry I
WS 09/10

Exercise Sheet 2

Exercise 1: Spherical trigonometry. (4 pts)

Given a spherical triangle with side lengths a, b, c and interior angles α, β, γ prove the following formula:

$$2 \sin \frac{A}{2} = \frac{\sqrt{\sin s \sin(s-a) \sin(s-b) \sin(s-c)}}{\cos\left(\frac{a}{2}\right) \cos\left(\frac{b}{2}\right) \cos\left(\frac{c}{2}\right)},$$

where $A = \alpha + \beta + \gamma - \pi$ and $s = \frac{1}{2}(a+b+c)$.

Exercise 2: Geometry. (4 pts)

Positions on the globe can be specified using longitude and latitude. Longitude is an east-west angle measured with respect to Greenwich, England. Latitude is a north-south measurement with respect to the equator. To the nearest degree, New York has coordinates $(74°W, 41°N)$ and Berlin has coordinates $(13°E, 52°N)$.
(a) What is the distance between these two cities? (The circumference of the earth is about 40,000 km).
(b) How close does the shortest path come to the North Pole?

Exercise 3: Diagonalization of a quadratic form. (4 pts)

Let $Q(\mathbf{x})$ be the quadratic form on $\mathbb{R}^3$ defined by $Q(\mathbf{x}) = x_0x_1 + x_1x_2 + x_2x_0$ for $\mathbf{x} = (x_0, x_1, x_2)$. Find a linear change of coordinates $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that with respect to the coordinates $\mathbf{y} = T(\mathbf{x})$, the quadratic form is diagonal: $Q(\mathbf{y}) = \sum \pm y_i^2$. Determine the rank and the signature of $Q(\mathbf{x})$.

Exercise 4: Reflection in a hyperplane. (4 pts)

Obtain the reflection in a hyperplane as the limit of inversions in hyperspheres for the radii going to infinity.

Due: Tutorial on 06.11.09