Geometry I

Practice problems for Test, 2009 December 17

1. Suppose four points \( A, B, C, D \in \mathbb{R}P^1 \) have cross-ratio \( R(A, B, C, D) = q \). What is the cross-ratio \( R(B, C, A, D) \)?

2. Let \( \triangle ABC \) be a hyperbolic triangle with angles \( \alpha = \pi/6, \beta = \pi/4, \gamma = \pi/2 \). What are the side lengths? What is the area of the triangle?

3. Suppose \( \ell, \ell' \) are lines in \( \mathbb{H}^2 \) with unit normal vectors \( n = (5/4, 0, 3/4) \) and \( n' = (1, 1, -1) \), respectively, in Lorentz space. Do these lines intersect? If so, at what angle and at what point \( p \in \mathbb{H}^2 \)? If not, what is the distance between the lines?

4. Show that there are circles in the hyperbolic plane which cannot be inscribed into a triangle.

5. Suppose \( U \) and \( U' \) are vector subspaces of \( V \). Show that the projective subspace \( P(U + U') \subset P(V) \) is the union of all lines \( xx' \) where \( x \in P(U) \) and \( x' \in P(U') \) with \( x \neq x' \).