1 Aufgabe

Let \( M = \mathbb{R}^2 \) and \( \theta : \mathbb{R} \times M \to M \) be given by the formula
\[
\theta_t(x, y) = (x \cos t + y \sin t, -x \sin t + y \cos t).
\]

- Show that \( \theta \) is a globally defined action of \( \mathbb{R} \) on \( M \).
- Describe \( X \), the associated infinitesimal generator.
- Describe the orbits.
- Show explicitly that \( X \) is invariant with respect to \( \theta \), i.e., that \( \theta_t^*(X_{(x,y)}) = X_{\theta_t(x,y)} \).

2 Aufgabe

Let \( M = \mathbb{R}^2 \), the \( x, y \) plane, and \( X = y \frac{\partial}{\partial x} + x \frac{\partial}{\partial y} \). Find the corresponding domain \( W \) and the local one-parameter action \( \theta : W \to M \).

3 Aufgabe

Consider the vector field \( X := x^2 \frac{\partial}{\partial x} \) on \( M := \mathbb{R} \). Find the local associated flow \( \theta \) and describe its domain \( W \).

4 Aufgabe

Let \( M = GL(2, \mathbb{R}) \) and define an action of \( \mathbb{R} \) on \( M \) by the formula
\[
\theta_t(A) := \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \cdot A, \quad A \in GL(2, \mathbb{R}),
\]
with the dot denoting matrix multiplication. Find the infinitesimal generator.