# Differentialgeometrie II
# Übungsblatt 1

October 22, 2008

1. Let $M$ and $N$ be $C^\infty$ manifolds of dimensions $m$ and $n$. Show that $M \times N$ is a $C^\infty$ manifold of dimension $m+n$ with coordinate charts of the form $\{U \times V, \phi \times \psi\}$, where $(U, \phi)$ and $(V, \psi)$ are coordinate charts for $M$ and $N$, respectively, and $\phi \times \psi(p, q) = (\phi(p), \psi(q))$ in $\mathbb{R}^{m+n}$.

   (Boothby, theorem III.1.1)

2. Let $M$ be a smooth manifold and $U \subset M$ open. Let $\phi : U \to R^n$. Then $(U, \phi)$ is a coordinate chart if and only if $\phi$ is a diffeomorphism onto an open subset $W \subset \mathbb{R}^n$. Analogously, let $W \subset \mathbb{R}^n$ be open, $\psi : W \to M$ a dippheomorphism onto an open subset $U \subset M$, then $(U, \psi^{-1})$ is a local chart.

3. View the two-dimensional torus as $S^1 \times S^1$, i.e. the product of two circles (considered as smooth one-dimensional manifolds). Use problem 1) to define a smooth structure on $S^1 \times S^1$.

4. Consider $\phi : \mathcal{B}_1^2 \to \mathbb{R}^2$, a map from the unit open ball $\mathcal{B}_1^2 \subset \mathbb{R}^2$ to $\mathbb{R}^2$, defined by $\phi((x_1, x_2)) = (\frac{x_1}{\sqrt{1-x_1^2-x_2^2}}, \frac{x_2}{\sqrt{1-x_1^2-x_2^2}})$. Show that $\phi$ is (at least) a $C^1$-diffeomorphism. Hint: $\phi^{-1}((y_1, y_2)) = (\frac{y_1}{\sqrt{1+y_1^2+y_2^2}}, \frac{y_2}{\sqrt{1+y_1^2+y_2^2}})$. If you can, prove that it is actually $C^\infty$.