Suppose n is a topological manifold with smooth structure $\mathcal{U}$. Suppose M is a topological manifold and F is a homeomorphism $M \to N$. We can use F to "pull back" the smooth structure $\mathcal{U}$ from N to M as follows: for each chart (U, ϕ) in $\mathcal{U}$ we have a chart $(F^{-1}(U), \phi \circ F)$ on M. These charts form a maximal smooth atlas $F^*(\mathcal{U})$ for M, that is a smooth structure on M. Almost by definition, the map $F\colon M \to N$ is a diffeomorphism from $(M, F^*(\mathcal{U}))$ to $(N,\mathcal{U})$.

Now consider the case $M = N$. If F is the identity map, then $F^*(\mathcal{U}) = \mathcal{U}$. More generally, this is true any time F is a diffeormorphism with respect to the smooth structure $\mathcal{U}$ (that is, a diffeomorphism from the smooth manifold $(M,\mathcal{U})$ to itself).

But suppose F is a homeomorphism which is *not* a diffeomorphism. Then $(N, F^*(\mathcal{U})) = (M, F^*(\mathcal{U}))$ is a *distinct* differentible structure on the same differentiable manifold M. For instance, a real-valued function $g\colon M \to \mathbb{R}$ is smooth w.r.t. $\mathcal{U}$ iff $g \circ F$ is smooth w.r.t. $F^*(\mathcal{U})$. But, as mentioned above, the map F is a diffeomorphism as a map $(M, F^*(\mathcal{U})) \to (M,\mathcal{U})$. In particular, the two smooth manifolds are really the same (the are diffeomorphic to each other).

Thus, this simple construction never produces "exotic" smooth structures (two different smooth manifolds with the same underlying topological manifold).

Facts:

(1) Up to diffeomorphism, there is a unique smooth structure on any topological manifold M^n in dimension $n \leq 3$. Up to diffeomorphism, there is a unique smooth structure on $\mathbb{R}^n$ for $n \neq 4$.

(2) The *Hauptvermutung* (with that name even in English) of geometric topology (formulated 100 years ago) suggested that every topological manifold should have a unique PL structure (given by a triangulation) and a unique smooth structure. This is now known to be false.

(3) Every smooth manifold has a triangulation. Not every topological M^4 has a triangulation.

(4) There are uncountably many smooth structures on $\mathbb{R}^4$. It is unknown if there is any exotic smooth structure on S^4.

(5) In higher dimensions, some things get easier. In dimensions $n \geq 7$, for instance, there are exotic spheres S^n, but these form a well-understood finite group (e.g., there are 28 for $n = 7$). In general, the differences between smooth and PL manifolds, and between PL and topological manifolds, can be analyzed for $n \geq 5$ by means of algebraic topology.