Topology WS 2006–07

Homework assignment 10, to be reviewed before the test on 24. Jan. 2007

These problems are not to be turned in.

(1) Let $P = \mathbb{S}^2/\pm$ be the projective plane. Let $U = P \smallsetminus \{p\}$ be the complement of a single point in P. (U is homeomorphic to a Möbius band.) What is $\pi_1(U)$?

(2) Suppose X is the figure-eight space, consisting of circles A and B joined at the basepoint x_0. Its fundamental group is

$$\pi_1(X, x_0) = \langle A \rangle \star \langle B \rangle \cong \mathbb{Z} \star \mathbb{Z}.$$

(a) Consider the infinite cyclic subgroup $H < \pi_1(X)$ generated by the element A^2. What is the covering space of X corresponding to this subgroup H? (Don't try to give a rigorous proof. Just sketch the space and explain why your sketch is correct.) What is the automorphism group of this cover?

(b) Now consider the map from $\pi_1(X)$ to the abelian group $\mathbb{Z}/3 \oplus \mathbb{Z}/3$ which takes A to $(1, 0)$ and B to $(0, 1)$. The kernel of this map is a normal subgroup K of π_1. What is the cover of X corresponding to this subgroup K? (Again, sketch the cover and explain why it corresponds to this kernel, without giving a full proof.) What is the automorphism group of this cover?