



Berlin
Mathematical
School



Topology

WS 2006–07

Homework assignment 8, due 20. Dec. 2006

- (1) Show that the fundamental group of the closed disk D^2 is trivial. Show that winding number provides an isomorphism from $\pi_1(\mathbb{S}^1, *)$ to \mathbb{Z} . Use this to give a new proof that there is no retraction from D^2 to \mathbb{S}^1 .
- (2) Show that
$$\pi_1(X \times Y, \{x_0, y_0\}) \cong \pi_1(X, x_0) \times \pi_1(Y, y_0).$$
- (3) Show that having the same homotopy type is an equivalence relation. Show that a homotopy equivalence $f : X \rightarrow Y$ induces an isomorphism f_* of fundamental groups. In particular if $i : X \rightarrow Y$ embeds X as a deformation retract of Y , then i_* is an isomorphism.
- (4) If $f : \mathbb{S}^n \rightarrow \mathbb{S}^n$ has no fixed points, show that f is homotopic to the antipodal map $p \mapsto -p$.