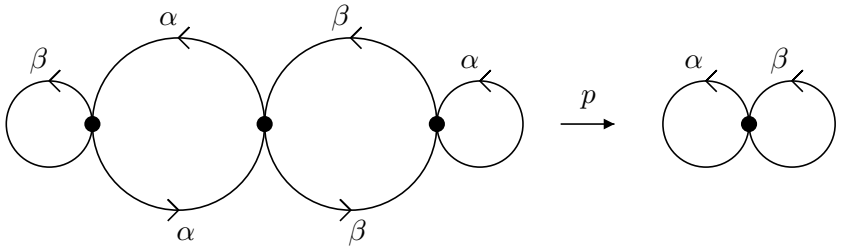
Berlin Mathematical School

Topology WS 2006–07

Homework assignment 6, due 6. Dec. 2006

(1) Suppose $p : Y \to X$ and $p' : Y' \to X$ are connected coverings, with X furthermore locally connected. Suppose there is a continuous map $q : Y \to Y'$ such that $p' \circ q = p$. Show that q is a covering map.

(2) Suppose X is the "wedge of two circles" and Y is the space consisting of four circles joined as in the figure. Let $p : Y \to X$ be the mapping taking each of the three arcs labeled α to the circle α, and similarly for β. Show that p is a three-sheeted covering. Let γ be the closed path in X given by $\alpha * \beta * \bar{\alpha} * \bar{\beta}$, that is, it follows α, then β, then α in reverse, then β in reverse. What are the three liftings of α to Y? Deduce that γ is not homotopic in X to a constant path. Show however that γ is homologous to zero.



(3) Let $I^n = [0,1]^n$ be an n-dimensional rectangle. Prove, using the Homotopy Lifting theorem and induction on n, that any covering of I^n is trivial.