Berlin Mathematical School

Topology WS 2006–07

Homework assignment 5, to review before the test on 29. Nov. 2006

These problems are not to be turned in.

(1) Suppose U and V are open sets in $\mathbb{R}^2$ with $U \cap V$ connected and $H_1(U \cap V) = 0$. Show that $H_1(U \cup V)$ is isomorphic to $H_1 U \oplus H_1 V$.

(2) Suppose A and B are disjoint closed subsets of $\mathbb{R}^2$. Show that $H_1(\mathbb{R}^2 \setminus (A \cup B))$ is isomorphic to $H_1(\mathbb{R}^2 \setminus A) \oplus H_1(\mathbb{R}^2 \setminus B)$. If $\mathbb{R}^2 \setminus A$ has m components and and $\mathbb{R}^2 \setminus B$ has n components, show that $\mathbb{R}^2 \setminus (A \cup B)$ has $m + n - 1$ components.

(3) Suppose $U \subset \mathbb{R}^2$ is open, and $K \subset U$ is compact. Show $H_1(U \setminus K) \cong H_1 U \oplus H_1(\mathbb{R}^2 \setminus K)$.