Quasiisothermic Mesh Layout

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joint work with
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Triangulated Surface
Optimized PQ-Mesh with touching incircles
More surfaces
From Discrete Differential Geometry

Planar quadrilaterals with touching incircles approximate conformal curvature line (*isothermically*) parameterized surfaces.

Was used before to create some minimal surfaces.
Curvature Lines and Conformal $\rightarrow$ Isothermic.
Isothermic Parameterization of a Triangle Mesh

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Quad-Panel Generation

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PQ/Incircle Optimization
Discrete Conformal Parameterization

A map to the plane that mimics the angle preserving property of smooth conformal maps.

Classical Riemann-Map to the Circle
Conformal maps with different boundary conditions.


ACM Transactions on Graphics 27:3 [Proceedings of ACM SIGGRAPH 2008]
Curvature Boundary Conditions
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Boundary angle sum $\theta$ at a boundary vertex
Curvature boundary conditions and a singularity with angle $540^\circ$
We call a conformal parameterization with curvature boundary conditions a *quasiisothermic parameterization*.
Quad-Mesh Generation
Variational Principle

Linear combination of energies

\[ E := \lambda_1 E_{\text{planar}} + \lambda_2 E_{\text{incircle}} + \lambda_3 E_{\text{touch}} \]

▷ \( E_{\text{planar}} \) is some planarity term

▷ \( E_{\text{incircle}} \) is due to A. Schiftner, M. Höbinger, J. Wallner, and H. Pottmann. 2009. *Packing circles and spheres on surfaces.* ACM Trans. Graph. 28

▷ \( E_{\text{touch}} \) critical for touching incircles
Touching Circles Energy

\[ E_{\text{touch}}(ij) = \left( \cot \frac{\beta_j}{2} \cot \frac{\beta_i}{2} - \cot \frac{\beta_i}{2} \cot \frac{\beta_j}{2} \right)^2. \]
Curvature Boundary Conditions

Optimization of the energy
Optimization of big meshes needs a good guess → Quasiisothermic parameterizations