Homothetic Triangle Contact Representations

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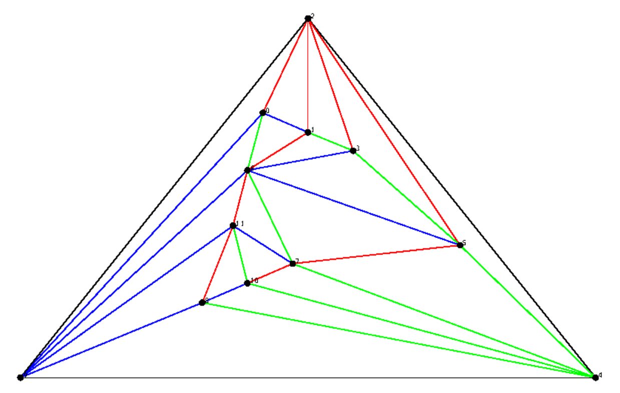
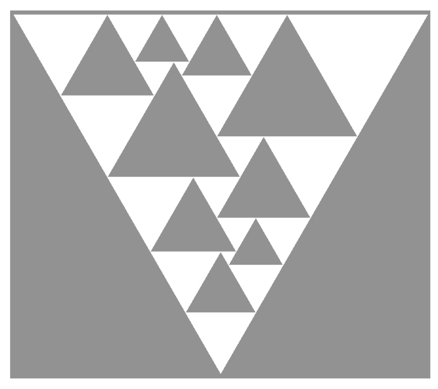


We reprove the existence of contact representations by homothetic triangles:
Theorem. *Let G be a 4-connected triangulation. Then there exists a contact representation of G by homothetic triangles.*

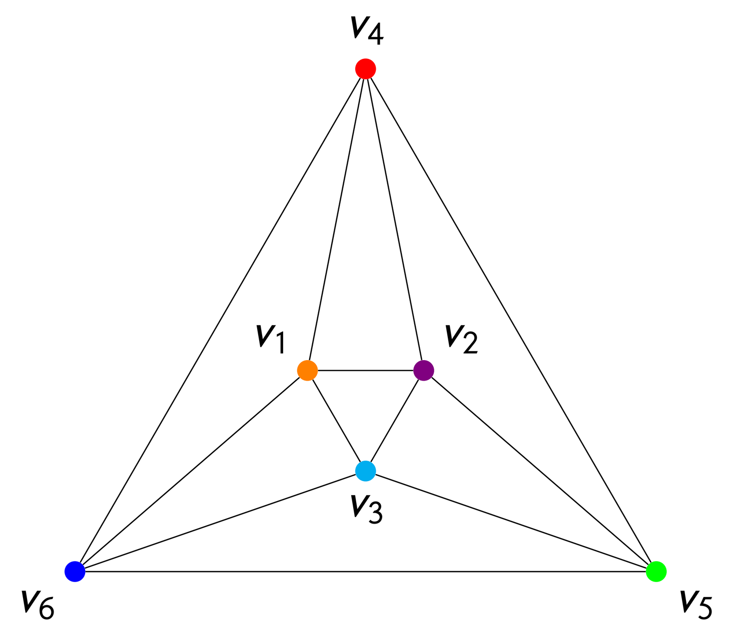


Generalization

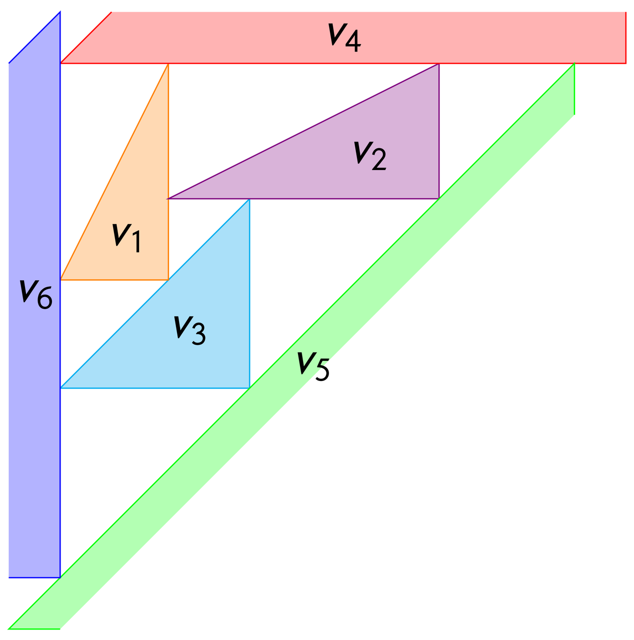
Theorem. *Let G be a 4-connected triangulation and $\tilde{r} \in \mathbb{R}^n_{>0}$. Then there exists a right triangle contact representation of G with aspect ratio vector $\tilde{r}$.*
The case $\tilde{r} = (1, \ldots, 1)$ is equivalent to the original theorem.

Example

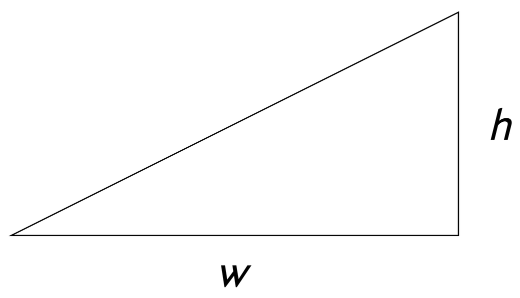
The given graph



A right triangle contact representation



The aspect ratio vector



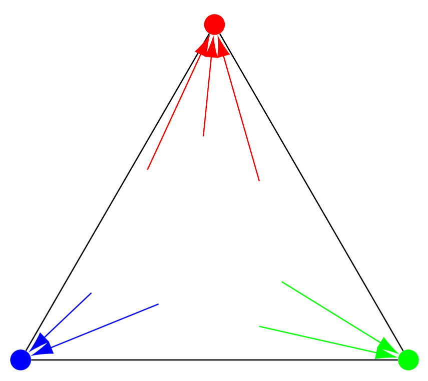
aspect ratio := $\frac{h}{w}$

aspect ratio vector of the example on the left
$= (2, 1/2, 1, 1, 1, 1)$

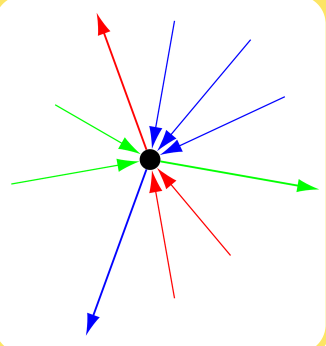
Schnyder Woods

Orientation and coloring of the inner edges of a triangulation fulfilling:

Outer vertex property

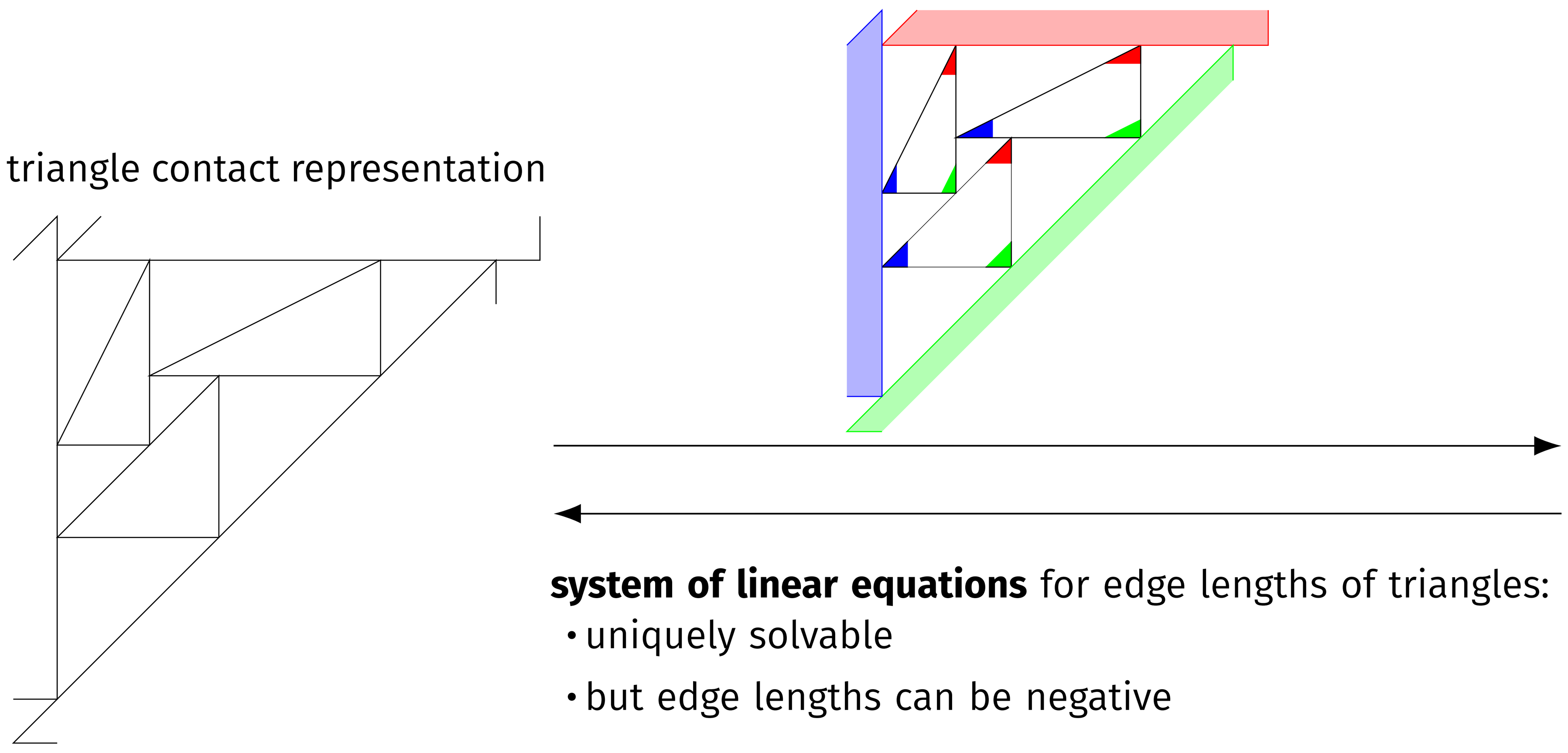


Inner vertex property

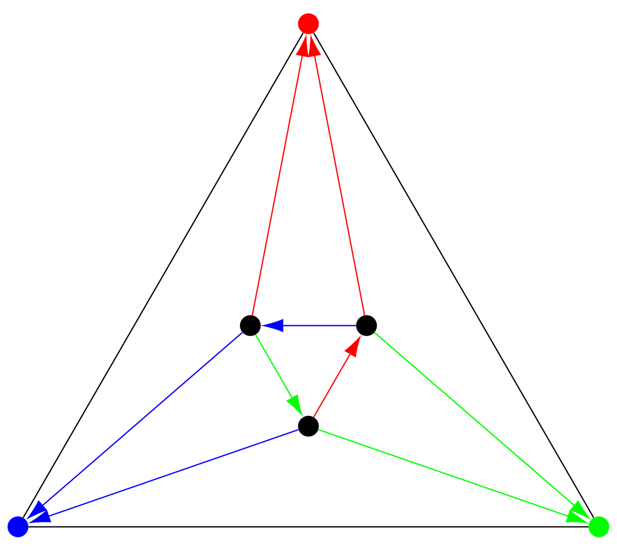


Induced Schnyder Woods

triangle contact representation



Schnyder Wood

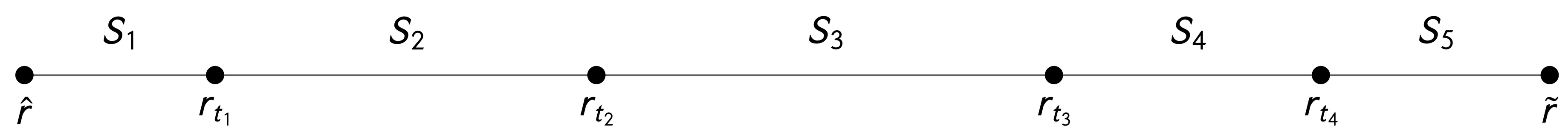


system of linear equations for edge lengths of triangles:
- uniquely solvable
- but edge lengths can be negative

We say that Schnyder Wood S *realizes* r if there is a contact representation with aspect ratio vector r that induces S.

Proof Strategy

- Start with arbitrary right triangle contact representation with aspect ratio vector $\hat{r}$ and Schnyder Wood S_1.
- Examine the line segment of aspect ratio vectors connecting $\hat{r}$ and $\tilde{r}$: $\{r_t := (1-t)\hat{r} + t\tilde{r} : 0 \le t \le 1\}$.
- By studying the system of linear equations, we see that S_1 realizes a subsegment $\{r_t : 0 \le t \le t_1\}$ and that there is a Schnyder Wood S_2 realizing a subsegment $\{r_t : t_1 \le t \le t_2\}$, and so on (in fact, we sometimes have to perturb the starting point $\hat{r}$).
- Then we show that a line segment can only be divided into a bounded number of subsegments in this manner. Thus $\tilde{r}$ has to be reached at some point.



Open Questions

- Can homothetic triangle contact representations be computed efficiently?
- Are homothetic triangle contact representations unique?

References

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[3] J. Rucker. "Kontaktdarstellungen von planaren Graphen". Diplomarbeit. Technische Universität Berlin, 2011. URL: page.math.tu-berlin.de/~felsner/Diplomarbeiten/dipl-Rucker.pdf.