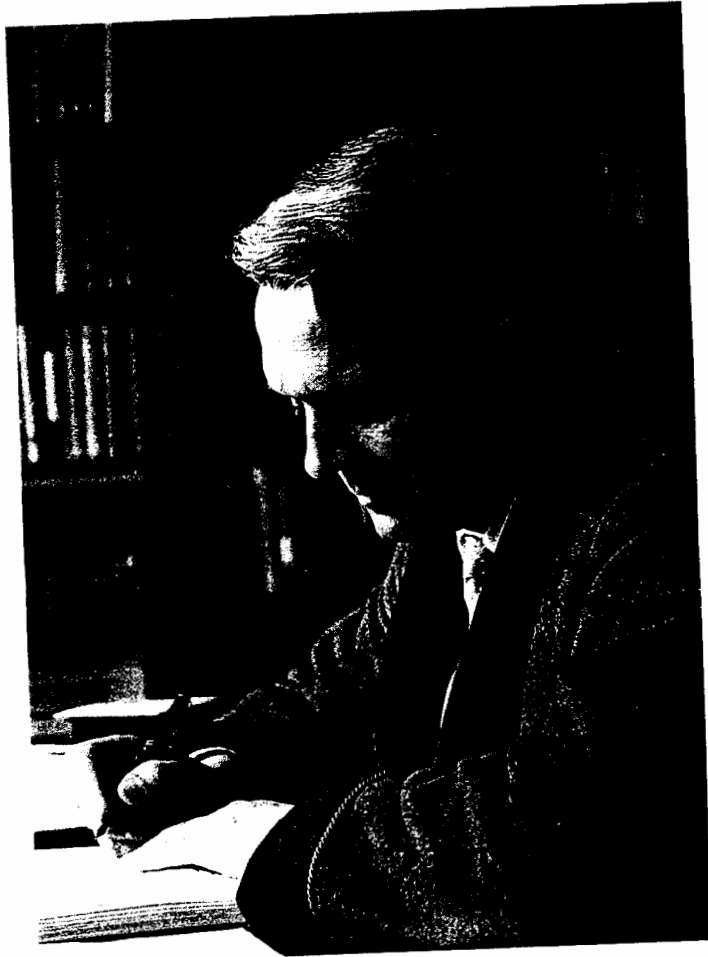


**CONTRASTING VIEWS
OF MATHEMATICS
Helmut Wielandt and
Jean Dieudonné, 1960**

by
Hans Schneider
June 2002



Helmut Wielandt around 1960

Philip Hall (1964)
on Wielandt's contributions to
finite group theory

"In this field he is the natural successor of Georg Frobenius and Issai Schur; and he is at least their equal. Indeed, in some respects, he is their superior, most notably in the air of easy elegance which he imparts to everything he writes. This mastery of style is something much deeper than surface polish; and his early papers remain today as classical statements, largely immune from the hand of the improver."

2

Wielandt's Inaugural Address
Heidelberger Akademie der
Wissenschaften
pub. 1961/62

1) The main line of development of mathematics has been characterized for several decades by the invasion of the axiomatic method into ever more areas. The goal is to derive all of mathematics deductively from just a few basic principles such as order and continuity. By turning increasingly towards the abstract, revolutionary unification has been achieved in mathematics, ...

It is as if some areas of mathematics which earlier could hardly be reached on foot are now connected by motorways.

In fact, the impetus which Göttingen had given to abstract algebra reached Berlin just when I was a student, and recognition of the implications of the axiomatic method fascinated me just as it did my fellow students. But I could not share the general opinion that this would henceforth be the only rewarding direction for research.

It seemed to me that, like all great deductive systems, it was threatened by the danger that the problems which it could not properly accommodate would be dismissed as uninteresting, whereas on the contrary, these ought to provide a stimulus to broaden the foundations. In terms of the metaphor I used earlier, this research area seems to me to be a mountainous region that is still undisturbed by roads and has to be traversed on foot. But this has its charm. And the nice surprises that one experiences compensate for the occasional pitying glances of motorists."

✓

6

Bourbaki, Elements of the history of mathematics, Chapter on Linear and Multilinear Algebra

“Thus we reach the modern era, where the axiomatic method and the notion of structure (felt at first, defined only recently), allow us to separate concepts that up till then had been inextricably mixed, to formulate that which was vague or subconscious, to prove in appropriate generality theorems which had been known only in particular cases.”

6

6a

ACTUALITÉS SCIENTIFIQUES ET INDUSTRIELLES

1032

ÉLÉMENTS DE MATHÉMATIQUE

PAR

N. BOURBAKI

VI

PREMIÈRE PARTIE

LES STRUCTURES FONDAMENTALES DE L'ANALYSE

LIVRE II

ALGÈBRE

CHAPITRE II

ALGÈBRE LINÉAIRE



1947
3

PARIS

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Jean Dieudonné



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JOC/EFR August 2001

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5/27/2002

Dieudonné's review of
Emil Artin's
"Geometric Algebra" (1957),
Math Rev 18, 553e

) For the last 20 years all mathematicians concerned with linear algebra have known how greatly the "intrinsic" language of geometry allows us to simplify and clarify this theory. Nevertheless, hardly a year passes when there do not get published one or two "textbooks" on "matrix calculus", most often of distressing mediocrity. They are dreary compilations of recipes of calculations without motivation or apparent interest.

✓

It is reassuring to see a mathematician as universally respected as Artin taking the trouble to show by example, in the most attractive way, how, beginning with the most rudimentary notions of algebra, one can guide the student up to most difficult problems in theory of classical groups. The experiment is a resounding success. One does not know what to admire the most, the perfect lucidity of the style, which is neither heavy nor pedantic, the mastery of the organization of lemmas and theorems, or the endless ingenuity of each demonstration.



A TREATISE

on the
Theory of Determinants

BY
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REVISED AND ENLARGED BY
WILLIAM H. METZLER, PH.D., F.R.S.E., F.R.S.C.

1933

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If in (2) we replace a_m by $2a_m$, then it becomes

$$\begin{aligned}
 & K \begin{pmatrix} b_1 \cdots b_{m-1} & c_{m-1} \cdots c_1 \\ a_1 & \cdots & 2a_m & \cdots & a_1 \\ c_1 \cdots c_{m-1} & b_{m-1} \cdots b_1 \end{pmatrix} \\
 & = K(1, m-1) [K(1, m) + a_m K(1, m-1) \\
 & \quad - b_{m-1} c_{m-1} K(1, m-2)] \\
 & = 2K(1, m)K(1, m-1).
 \end{aligned}
 \tag{3}$$

558. A continuant of even order may be transformed by alternately multiplying and dividing the successive elements of the main diagonal by any given number; and a continuant of odd order may be so transformed if in addition the continuant be divided by this number.

This is readily seen on performing the following operations: multiply $\text{row}_1, \text{row}_3, \dots$, by k , then divide $\text{col}_2, \text{col}_4, \dots$, by k . Thus

$$\begin{vmatrix} a_1 & 1 & \cdot & \cdot \\ c_1 & a_2 & b_2 & \cdot \\ \cdot & c_2 & a_3 & b_3 \\ \cdot & \cdot & c_3 & a_4 \end{vmatrix} = \begin{vmatrix} ka_1 & b_1 & \cdot & \cdot \\ c_1 & \frac{a_2}{k} & b_2 & \cdot \\ \cdot & c_2 & ka_3 & b_3 \\ \cdot & \cdot & c_3 & \frac{a_4}{k} \end{vmatrix}$$

$$\begin{vmatrix} a_1 & b_1 & \cdot & \cdot & \cdot \\ c_1 & a_2 & b_2 & \cdot & \cdot \\ \cdot & c_2 & a_3 & b_3 & \cdot \\ \cdot & \cdot & c_3 & a_4 & b_4 \\ \cdot & \cdot & \cdot & c_4 & a_5 \end{vmatrix} = \frac{1}{k} \begin{vmatrix} ka_1 & b_1 & & & \\ c_1 & \frac{a_2}{k} & b_2 & & \\ \cdot & c_2 & ka_3 & b_3 & \\ \cdot & \cdot & c_3 & \frac{a_4}{k} & b_4 \\ \cdot & \cdot & \cdot & c_4 & ka_5 \end{vmatrix}$$

The multiplications and divisions by k may stop before the last a is reached or they may not begin until the $(2h-1)$ st row is reached and give corresponding results.

559. The continuant $K(1, n)_d$ equals

$$\frac{d_{n-1}d_{n-3}d_{n-5} \cdots}{d'_{n-1}d'_{n-3}d'_{n-5} \cdots}$$

Express the square of any even ordered determinant as a zero-axial skew of the same order.

PERSYMMETRIC DETERMINANTS

444. A determinant such that each line perpendicular to the principal diagonal has all its elements alike is called a *persymmetric* determinant. In the persymmetric determinant of the n th order

$$\begin{vmatrix} a_1 & a_2 & a_3 & \cdots & a_n \\ a_2 & a_3 & a_4 & \cdots & a_{n-1} \\ a_3 & a_4 & a_5 & \cdots & a_{n-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_n & a_{n-1} & a_{n-2} & \cdots & a_{2n-1} \end{vmatrix}$$

there are evidently at most $2n-1$ distinct elements, viz. those of the principal diagonal and one adjacent minor diagonal. It may thus be shortly denoted by

$$P(a_1 a_2 \cdots a_{2n-1}).$$

It is apparent from the nature of a persymmetric determinant that, using double suffix notation, any minor is not altered if we add (or subtract) the same integer to each row-number provided we subtract (or add) it from the column numbers.

445. The persymmetric determinant of $a_1, a_2, \cdots, a_{2n-1}$, is equal to the persymmetric determinant of $a_1, ma_1+a_2, m^2a_1+2ma_2+a_3, m^3a_1+3m^2a_2+3ma_3+a_4$, etc.

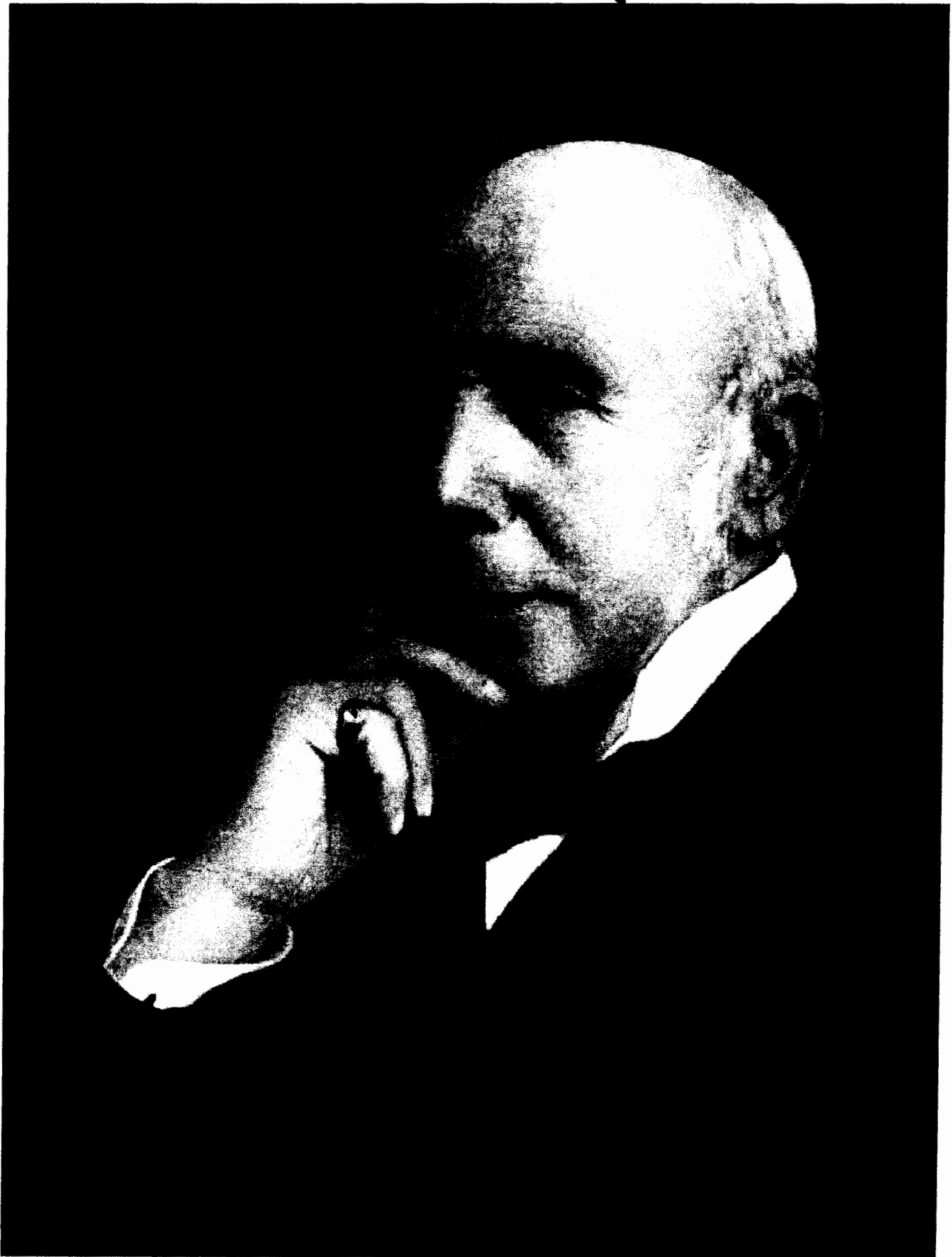
This follows from multiplying the determinant row-wise by

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \cdots \\ m & 1 & 0 & 0 & 0 \cdots \\ m^2 & 2m & 1 & 0 & 0 \cdots \\ m^3 & 3m^2 & 3m & 1 & 0 \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}$$

and repeating the operation upon the product.

Indicating row multiplication as practised in the multiplication of determinants by χ ,—for example, writing $(a, b, c \chi \alpha, \beta, \gamma)$ for $a\alpha+b\beta+c\gamma$,—the elements of the new persymmetric determinant here found are very conveniently denoted by $a_1, (a_1, a_2 \chi m, 1), (a_1, a_2, a_3 \chi m, 1)^2, (a_1, a_2, a_3, a_4 \chi m, 1)^3$ etc.

THOMAS MUIR



Array Algorithms

for

STRUCTURED MATRICES

THOMAS KAILATH

Dept. of Elect. Eng.
Stanford University

(Joint work with T. Boros, B. Hassibi, H. Lev-Ari and A. Sayed)

Elementary Circular or Givens Rotations

- We can also employ elementary rotations that annihilate one entry at a time. They offer more flexibility, albeit at 20% increase in cost.
- An elementary 2×2 unitary rotation

$$\Theta = \frac{1}{\sqrt{1 + |\rho|^2}} \begin{bmatrix} 1 & -\rho \\ \rho^* & 1 \end{bmatrix} \quad \text{where } \rho = \frac{b}{a}, \quad a \neq 0,$$

performs the transformation

$$\begin{bmatrix} a & b \end{bmatrix} \Theta = \begin{bmatrix} \pm e^{j\phi_a} \sqrt{|a|^2 + |b|^2} & 0 \end{bmatrix}.$$

- In the trivial case $a = 0$, we can use the permutation matrix,

$$\Theta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Nicholas J. Higham

University of Manchester
Manchester, England

Accuracy and Stability of Numerical Algorithms

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The end of Wielandt's Inaugural Address

» But it is unmistakable that questions about finite structures are again coming strongly to the fore also in other areas of mathematics, partly as a result of growing applications of computing machines. I am convinced that the “finite” direction will be reunited with the mainstream in the course of the next few decades.“

THE [REDACTED] 77

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