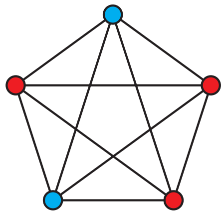
Bachelor / Master Thesis

Sparse Ramsey graphs

Description. Suppose we want to color the vertices of a given graph G with two colors (red and blue) such that G does not contain a monochromatic triangle (a triangle where all three vertices are of the same color). It is intuitively clear that if G has only few edges (on the given number of vertices), then such a coloring exists, whereas if G has many edges, it will be difficult or even impossible to find such a coloring. E.g., any coloring of the vertices of the complete graph on five vertices contains a monochromatic triangle:



More formally, we define for any graph G the *density of* G as $m(G) := \max_{H \subseteq G} e(H)/v(H)$, where the maximimization is over all subgraphs H of G. Moreover, for any graph F (above we considered only the special case of a triangle, $F = K_3$) we define the *Ramsey density* of F as

$$m^*(F) := \inf\{m(G) \mid \text{any two-coloring of } G \text{ contains a monochromatic copy of } F\} .$$

In [1], the authors determine the value of $m^*(F)$ for cliques $F = K_\ell$ on ℓ vertices and prove bounds for stars $F = S_\ell$ with ℓ rays. The simplest graph F for which the Ramsey density $m^*(F)$ is not known precisely is $F = S_2$. The best known bounds for this case are

$$\frac{4}{3} \leqslant m^*(S_2) \leqslant \frac{7}{5}$$

from [1], where a prize money is offered for any improvement upon these bounds.

Goal. The goal of this thesis is to improve the known bounds for $m^*(F)$ for small graphs F (such as small trees or cycles). If time permits, the same question can be considered for the edge-coloring variant of the problem (cf. [2] and [3]), where edges instead of vertices have to be colored.

Prerequisites. Solid knowledge in graph theory and possibly some programming skills.

References.

[1] A. Kurek and A. Ruciński. Globally sparse vertex-Ramsey graphs, J. Graph Theory 18 (1) (1994) 73–81.

[2] A. Kurek and A. Ruciński. Two variants of the size Ramsey number, Discuss. Math. Graph Th. 25 (2005) 141–149.

[3] T. Mütze and U. Peter. On globally sparse Ramsey graphs, Disc. Math. 313 (2013) 2626–2637.

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