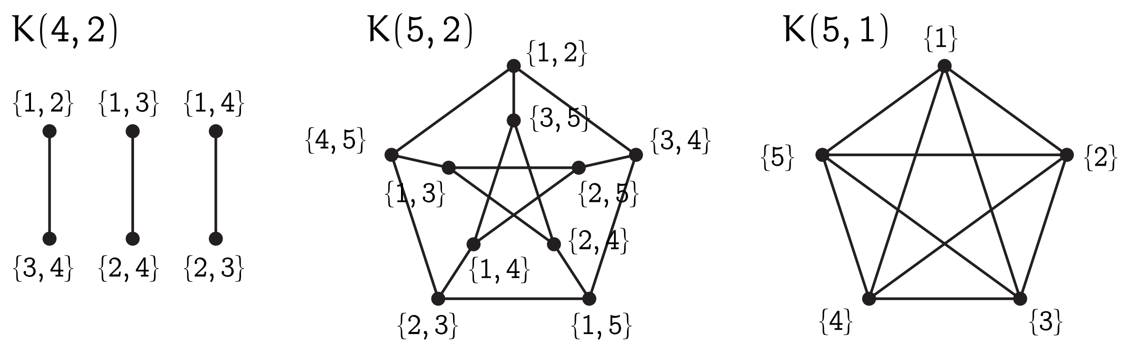
Bachelor / Master Thesis

Hamilton cycles in Kneser graphs

Description. The *Kneser graph* $K(n,k)$ has as vertices all k-element subsets of an n-element set, where any two disjoint sets are connected by an edge. Note that $K(5,2)$ is the well-known Petersen graph and $K(n,1)$ is the complete graph on n vertices. The figure below shows three small Kneser graphs:



Kneser graphs are highly symmetric in the sense that they 'look the same' from the point of view of any vertex (formally, these graphs are *vertex-transitive*). It has long been conjectured that apart from the Petersen graph $K(5,2)$, all connected Kneser graphs $K(n,k)$ (those for which $n \geqslant 2k+1$) have a Hamilton cycle, i.e., a cycle that visits every vertex exactly once. So far, this conjecture has been verified for all $n \leqslant 27$ and all relevant values of k [1], and for all n and k satisfying $n \geqslant 3k$, i.e., for the denser of the Kneser graphs [2]. For sparse connected Kneser graphs, in particular for $K(2k+1,k)$, the conjecture is still wide open (the graph $K(2k+1,k)$ is also known as the *odd graph*). In a recent breakthrough, we showed that a close relative of $K(n,k)$ indeed has a Hamilton cycle for all $k \geqslant 1$ and $n \geqslant 2k+1$ [3, 4].

Goal. The goal of this thesis is to extend the methods from [3, 4] and to prove new Hamiltonicity results for Kneser graphs and their relatives. In the explorative phase, this work may involve computer experiments (note that the number of vertices of $K(n,k)$ grows rapidly, so drawing small examples does not lead very far), but eventually we are interested in rigorous proofs.

Prerequisites. Basic knowledge in graph theory and possibly some programming skills.

References.

[1] I. Shields and C. Savage. A note on Hamilton cycles in Kneser graphs. *Bull. Inst. Combin. Appl.*, 40:13–22, 2004.
[2] Y. Chen. Kneser graphs are Hamiltonian for $n \geqslant 3k$. *J. Combin. Theory Ser. B*, 80(1):69–79, 2000.
[3] T. Mütze. Proof of the middle levels conjecture. *arXiv:1404.4442*, May 2014.
[4] T. Mütze and P. Su. Bipartite Kneser graphs are Hamiltonian. *arXiv:1503.09175*, June 2015.

Contact.

Torsten Mütze, MA 520, Tel. 030-314-78656, `muetze@math.tu-berlin.de`