Bachelor / Master Thesis

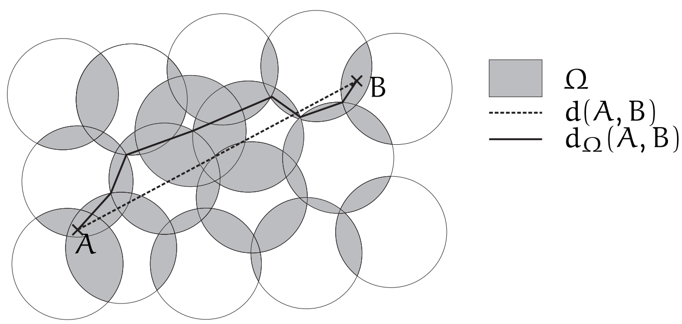
Shortest paths in disk coverings

Description. Suppose you are travelling from town A to town B with a mobile communication device. To ensure you never lose connection, you decide to travel only in regions that are within the transmission range of at least *two* base stations (each base station has a circular transmission range). The question is: How much of a detour do you have to accept compared to a direct route between A and B?

More formally, the problem can be stated as follows: Given a covering of the plane with (closed) unit disks, we define Ω as the region given by all points that are covered by at least two disks (the grey-colored region in the figure below). Given two points $A, B \in \Omega$, we let $d(A, B)$ denote the (direct) distance between A and B, and $d_\Omega(A, B)$ the length of a shortest path between A and B that uses only points from Ω. It was conjectured that for any choice of A and B we have

$$d_\Omega(A, B) \leqslant \sqrt{2} \cdot d(A, B) + c \ ,$$

where c is a small constant [1].



Goal. The goal of this thesis is to tackle this conjecture by implementing a given proof strategy (this should at least allow us to prove a constant which is reasonably close to $\sqrt{2}$, which would already be progress, cf. [2]).

Prerequisites. Good geometric intuition, willingness and endurance to translate intuition into formal proofs.

References.

[1] D. Baggett, A. Bezdek. On a shortest path problem of G. Fejes Tóth, Discrete Geometry, ed. A. Bezdek (2003), 19–26.

[2] E. Roldán-Pensado. Paths on the doubly covered region of a covering of the plane by unit discs, manuscript.

Contact.

Torsten Mütze, MA 520, Tel. 030-314-78656, muetze@math.tu-berlin.de