Stopwatch used during the UMTS-auction[1]

Auctions and the German UMTS-Auction

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The German UMTS-Auction in August 2000 attracted, like similar auctions in Great Britain and The Netherlands earlier in the year 2000, a lot of public attention and money. At this auction 4 to 6 licenses to offer third generation cell-phone service in Germany were on the block to be sold, raising a total revenue of 98.8072 BDM. In the following we outline the mathematics underlying the design of auctions and the light it sheds on the German UMTS-Auction. (Here UMTS is short for Universal Mobile Telecommunications System.)

A useful starting point is the auction organized by the *Regulatory Authority for Telecommunications and Posts* (RegTP)[2] to sell the stopwatch which was used during the UMTS-auction. The proceeds of the auction go to charity.[3]

RegTP used a first-price sealed-bid auction. In this auction bidders submit sealed bids. After the deadline, the auctioneer awards the object to the bidder with the highest bid who then pays what he bid. There are two questions one can ask: how should one bid and what is the revenue that the auctioneer will obtain? The first question requires a model of how bidders value the object and their attitude to risk. The benchmark model assumes that each bidder's monetary value for the stopwatch is an independent draw from a commonly known distribution F, say, and that bidders are risk neutral.[4] Uncertainty about bidders' valuations is needed to make the problem interesting. If bidders' valuations were publically known, the seller would approach the bidder with the highest valuation and make a 'take it or leave it' offer. The independence assumption rules out the possibility that a bidder's valuation may depend on another bidder's valuation or some common exogenous factor. For example, values for the stopwatch may depend on an estimate of its resale possibilities.

For simplicity assume F is the uniform distribution on $[0,1]$ and there are only two bidders. Let v be the value that bidder 1 assigns to the clock and w the value that bidder 2 assigns to the clock. What bid should bidder 1 submit? The first thought is to submit v itself (bidding truthfully). Might submitting a bid $u \neq v$ generate even higher expected payoff? For any bid there are two possibilities: winning the watch at bid u or not. If $P(u)$ is the probability of winning at bid u, the expected payoff is $(v-u)P(u)$. Thus the optimal bid u maximizes $(v-u)P(u)$. But what is $P(u)$? It must depend on bidder 2's bid and that bid will depend on bidder 1's bid. We can exit this logical impasse by treating the auction as a game of incomplete information amongst the bidders and determine the equilibrium of the game. In this context a strategy will be a function b that maps valuations into bids. If bidder i uses a bid function b_i then bidder 1's expected payoff will be:

$$(v - b_1(v)) \int_0^1 P(b_1(v) > b_2(t))\, dt.$$

An equilibrium of this game will be a pair of bid functions, b_1^* and b_2^* such that

$$\begin{aligned}(v - b_1^*(v)) &\int_0^1 P(b_1^*(v) > b_2^*(t))\, dt \\ &\geq (v - b(v)) \int_0^1 P(b(v) > b_2^*(t))\, dt \;\forall b \neq b_1^* \qquad (1)\end{aligned}$$

as well as a similar condition for bidder 2. How do we find such an equilibrium? Assume there exists an

1 All rights reserved. With kind permission of Pleiderer Aktiengesellschaft, `www.pfleiderer.de`
2 Regulierungsbehörde für Telekommunikation und Post `http://www.regtp.de`
3 See `http://www.regtp.de/aktuelles/pm/00131/index.html`.
4 That is bidders evaluate consequences according to expected monetary value.

equilibrium b^* where $b_1^* = b_2^* = b^*$ and b^* is strictly increasing. Then use the equilibrium condition to derive an equation for b^* which one then solves. Consider the equation (1) again. It can be rewritten to read:

$$b^* \in \arg\max_b\{[v - b(v)] \int_0^1 P(b(v) > b^*(t))\, dt\}.$$

By strict monotonicity of b^* we can for each v find an x such that $b^*(x) = b(v)$, thus:

$$v \in \arg\max_x (v - b^*(x)) \int_0^1 P(b^*(x) > b^*(t))\, dt.$$

However, $P(b^*(x) > b^*(t)) = P(x > t) = x$ since t is uniformly distributed. Thus to find b^* we focus on $\max_x (v - b^*(x)) \int_0^1 x\, dt$. The first order condition for optimality implies that

$$(b^*)'(v) = \frac{(v - b^*(v))}{v}.$$

This differential equation with boundary condition $b^*(0) = 0$ has the solution

$$b^*(v) = E[t | t \leq v] = v/2.$$

Hence, in equilibrium, each bidder behaves as if his valuation is the highest of the two and computes the expected value of the second highest valuation. This estimate of the second highest valuation is his bid. Thus each bidder *shades* his bid downwards from his true valuation by 50 %. The revenue to the seller will be $E(\min\{v, w\})$.

The analysis just described is typical. The rules of the auction combined with the definition of equilibrium are used to identify differential equations that equilibrium bid functions must satisfy. In more general cases one gets partial differential equations that do not appear to admit closed form solutions.

Returning to our clock story, it is natural to ask if the auctioneer could have raised more revenue by using a different auction. For example, suppose he had used a second price sealed bid auction. In such an auction the highest bidder wins the object but pays the second highest bid. Sometimes called a Vickrey auction in honor of the Nobel Laureate William Vickrey it has an interesting property which we will now describe. How should one bid in a Vickrey auction? The crucial observation is this: ones bid does not determine what one pays. Thus bidding below ones valuation reduces the probability of winning and does not change what one pays.[5] Thus, no matter what others bid, one is always better off bidding truthfully.[6] In this auction everyone bids truthfully and the auctioneer gains $\min\{v, w\}$. In expectation the second price sealed bid auction fetches the same revenue as the first price sealed bid auction. Would making the auction open rather than sealed bid make a difference? A popular open auction is the English ascending auction. So as to abstract away from the details of how the auctioneer behaves and minimum bid increments, one can imagine the auction going like this: a monitor shows a steadily rising price. As the price rises bidders can choose to stay in or drop out. The auction stops once only one bidder remains. Under the benchmark model this auction is strategically equivalent to the previous auction and so yields the same expected revenue. In fact, under the benchmark model all auctions that award the object to the bidder with the highest valuation and never penalize the lowest bidder generate the same expected revenue. If there are n bidders, and their valuations are ordered

$$v_{(1)} \geq v_{(2)} \geq v_{(3)} \geq \cdots \geq v_{(n)},$$

the expected revenue will be $E(v_{(2)})$, i.e., the expected value of the second highest order statistic. This is the *revenue equivalence* theorem (Myerson 1981).[7] The auction that maximizes expected revenue to the seller (in this model) does not satisfy the conditions of the revenue equivalence theorem. A rough description is as follows. If F is the distribution of valuations determine the x that maximizes $x(1 - F(x))$. Call it x^*. Now run a sealed bid auction where the highest bidder wins. Let b be the highest bid and z the second highest bid. If $b \geq \max\{x^*, z\}$, charge the winner $\max\{x^*, z\}$. Otherwise the object is not sold. This auction is a Vickrey auction with a reserve price of x^*. No winner can get the object for a price less than x^*. The seller boosts expected revenue with a floor on the selling price. This is like the seller bidding on the object increasing the competitive pressure on the buyers. But there is a cost. With positive probability the object may go unsold if no bid exceeds x^*; which is why the revenue equivalence theorem does not hold. Thus, as always, higher revenues come with higher risks. As the number of bidders gets large the reserve price x^* loses its bite. This is because as n gets large, the second highest bid will be $v_{(2)}$ which exceeds x^* with high probability. Hence under the benchmark model we are back to where we were before. Any reasonable auction will generate on average the same revenue. Thus the problem of choosing between auction forms only becomes

5 You can easily convince yourself that bidding above ones valuation does not help either.

6 This is of course only true under our assumption, that bidders do not care about the price others pay.

7 In fact, the revenue equivalence theorem holds under broader conditions.

interesting in a world that departs from the benchmark model. We consider these departures in the next section. We have focused on revenue maximization as the goal of the auctioneer. This is not the only objective an auctioneer can have. Another criterion is that of *economic efficiency*. If the object to be auctioned off is a local utility or fishing rights we may wish it to go to the agent who will use the asset most efficiently. This is operationalized as awarding the object to the person who values it the most. The sealed bid first price auction while not revenue maximizing is efficient in this sense. The revenue maximizing auction is not efficient because there is a positive probability that the object will not be allocated to anyone. One simple, yet important insight from the analysis is that no auction can guarantee to extract a revenue equal to the highest bidder valuation. The winning bidder will always walk away with some profit. Hence the fact that a winner was prepared to pay more than his bid is not a flaw of the auction. It is a fact of life. One must pay people to tell the truth. We close this section by noting that RegTP sold the stopwatch finally for 30 000 DM to the Pfleiderer AG.

Benchmark and beyond

The benchmark model of auctions assumes the following:

1. Risk neutral bidders known to seller.
2. Independent private values: bidders valuations are independent random variables and is private to that bidder alone.
3. Homogeneous bidders in the sense that all bidders valuations are drawn from the same distribution.
4. A single unit for sale.
5. What happens after the auction does not influence what happens in the auction. This rules out, for example, an active aftermarket in the object being traded.

If one replaces risk neutrality of bidders by risk aversion, the revenue equivalence theorem does not hold. The sealed bid auction generates a higher expected revenue than the open ascending or second price auction. The intuition here is that risk averse bidders are prepared to pay to increase the probability of winning. Now suppose bidders are heterogenous. For example, some bidders valuations are in the interval from 5 to 10 while others are in the interval 0 to 5, say. Revenue equivalence theorem does not hold in the heterogenous situation. In particular the expected revenue for the open ascending auction can be higher or lower than the expected price from the sealed bid auction. If it is possible to classify bidders into types (say wear polyester, drive a Mercedes) then the optimal auction uses a different reserve price for each type of bidder. Furthermore it 'favors' bidders with low valuations. That is the winner is not necessarily the highest bidder. The idea is that by favoring low valuation bidders you encourage high valuation bidders to bid more aggressively. The reader interested in a fuller discussion of these matters is referred to Klemperer (1999). Here we focus on items 2 and 4 because of their relevance to the UMTS-auction.

Correlated and interdependent values

If bidders values are correlated but still private the revenue maximizing auction in this environment leaves all bidders with zero profit. Correlation benefits the auctioneer because each bidder's bid reveals not only something about his valuations but the valuations of others. Of more relevance is the case when valuations are interdependent. The most studied case is called common values. A common values auction is one where the prize has an objective value common to all bidders. However, this value is not known with certainty. A simple example would be one where the prize to be auctioned off is a lottery ticket. The ticket pays 1 000 DM if an election results in a Christian Democrat and 0 otherwise. Disagreements between bidders about the value of the lottery ticket are not a matter of taste but of information. You and I will have different estimates of the probability that the ticket will pay off because of access to different information. The information that bidder i has about the value of the object is usually called bidder i's signal and let us denote it s_i. One interprets high values of s_i as suggesting high values of the object and vice-versa. The value to bidder i of the object is a function of both i's signal and the signals of the other bidders. A special case occurs when the value to bidder i increases in both his own signal and in the signal of other bidders. Thus good news for one is good news for all. In such an environment an open ascending auction generates a higher expected revenue than a sealed bid auction. If you see your rivals bidding aggressively it must be because they have a high signal. This suggests they possess good news. Knowing this, one should revise ones own estimate of the value of the prize. In a sealed bid auction, the information that others have is not revealed to others. To see why the common values case is relevant for the UMTS-auction, suppose there were just one spectrum licence on the block. The value of the licence is a function of the traffic that will travel on that portion of the spectrum. Bidders have different estimates of this traffic volume based on different information sources. An open auction will reveal whether a bidder's estimate is on the high or low side. While there is a common values aspect to the UMTS-auction it is far from being a pure common values environment. The value of the licence also depends on how the relevant portion of spectrum is priced and managed by the owner. These are things that are idiosyncratic to each bidder, introducing a private values component. This is not the only complication. In the UMTS-auction there are multiple licences to be auctioned. Thus the value of a licence will also depend on the competition between eventual winners of the auction. This means bidders care not just about what they win but what their rivals win (see [4] for the underlying mathematical problems).

Multiple objects

When auctioning multiple objects the most obvious question is whether to auction them off separately or all at once. Answers to this and similar questions vary depending on how one models bidders preferences.

The simplest case involves multiple units of the same object, each bidder wanting at most one unit and independent private values. The Vickrey auction can be generalized to this case. Bidders submit bids, and the K, say, objects are awarded to the K highest bidders, all of whom pay the highest losing bid. Like the Vickrey auction it results in the efficient outcome and no bidder benefits by bidding below his valuation. A revenue equivalence theorem holds in this case as well. A broad class of auctions generate the same expected revenue. This equivalence holds even when the K objects are sold at once or sequentially. The revenue maximizing auction in this environment does essentially the same thing but uses a reserve price. In this case it is easier to see why. Suppose there were 5 bidders for 4 objects. Since the 'supply' of units is close to the 'demand' for units, an auction will not bring in a large sum. By instituting a reserve price, the auctioneer prevents winning at a low price.[8]

The next level of complexity is multiple units of the same objects with some bidders wanting 2 or more units. In this model one must specify how each bidder values their first unit, their second unit and so on. Thus a bidders valuation is now a vector. An extension of the sealed bid Vickrey auction to this case is known. Unlike the previous case this extension requires different bidders to pay different prices for the same number of units. At first glance this is odd, but it is essential. To explain why we consider an 'intuitive' auction for selling multiple units of the same object. The auctioneer raises the price per unit (in a continuous way) and asks bidders to state how many units they want at the current price. If the sum total of requests exceeds the available supply, the auctioneer continues to raise the price. As the price rises, bidders lower their demands. Eventually a price is reached where the available supply equals the demand. The problem with such an auction is that it gives bidders an incentive to understate their demand. By reducing the amount demanded at any price, a price rise can be arrested. While a bidder suffers by getting fewer units this will be compensated for by a lower price per unit. This phenomenon is known as demand reduction. It is commonly associated with auctions of multiple units that have all winning bidders pay the same price per unit. Ausubel and Cramton (1998) contains a fuller discussion with examples.

With some exceptions, less is known about how to auction off heterogenous objects. In this case bidders attach values to different combinations of objects. A pair of objects that complement each other in the eyes of one bidder may be substitutes in the eyes of another. The difficulty is to design the auction so as to allow the bidders the flexibility to express their preferences over different subsets of items. Once they have expressed their preferences over subsets, finding the optimal way to divide the objects amongst the bidders becomes an integer programing problem. The reader is directed to [2] for a recent survey on these matters. To get a sense of the difficulties imagine one must auction off a dining room set consisting of one table and four chairs. Some bidders want the entire set, others just the table and others just a chair or two. Should one auction off each part of the set separately? What about the entire set as a single object? What about two separate auctions, one for the table and the other for a lot of four chairs? For the mathematical difficulties underlying the design problem see [4]. The US auction for spectrum rights was an instance of such an auction of heterogenous objects. In that case a simultaneous ascending auction (SAA) was proposed. Each object was sold in its own ascending auction. The auctions were run simultaneously and bidders were allowed to participate simultaneously in as many of them as they wished. Such a design suffers two problems. The first is called the *exposure problem.* Bidders pay too much for individual items or bidders with preferences for certain combinations drop out early to limit losses. As an illustration consider an extreme example of a bidder who values the combination of goods i and j at \$100 but each separately at \$0. In the SAA, this bidder may have to submit high bids on i and j to be able to secure them. Suppose that it loses the bidding on i. Then it is left standing with a high bid j which it

8 Lack of reasonably high reserve prices explains the small revenue generated by the Dutch and Swiss auctions in the summer 2000.

values at zero. The second, not unique to the SAA is called the *threshold problem.* Suppose three bidders, 1, 2 and 3 and two objects $\{x, y\}$. Let $v^i(S)$ be the value that bidder i assigns to the set S of objects. Suppose:

$$v^1(x,y) = 100,\ v^1(x) = v^1(y) = 0,$$
$$v^2(x) = v^2(y) = 75,\ v^2(x,y) = 0,$$
$$v^3(x) = v^3(y) = 40,\ v^3(x,y) = 0.$$

Notice that the bid that bidder i submits on the set S need not equal $v^i(S)$. If the bidders bid truthfully, the auctioneer should award x to 2 and y to 3, say, to maximize his revenue. Notice however that bidder 2 say, under the assumption that bidder 3 continues to bid truthfully, has an incentive to shade his bid down on x and y to, say, 65. Notice that bidders 2 and 3 still win but bidder 2 pays less. This argument applies to bidder 3 as well. However, if they both shade their bids downwards they can end up losing the auction. The problem arises because a collection of bidders whose combined valuation for distinct portions of a subset of items exceeds the bid submitted on that subset by some other bidder. It may be difficult for them to coordinate their bids to outbid the large bidder on that subset.

Design of the UMTS-auction

We now describe the rules of the recent UMTS-auction.[9] The auction involved two stages, the second of which was to sell the left over items from the first stages plus auxiliary unpaired spectrum. Here we focus on the first stages. In the first stages twenty-four items of 5 MHz spectrum were to be auctioned. They were bundled together into 12 blocks of paired spectrum.[10] In order to obtain a licence a bidder had to win at least two blocks and no bidder was allowed more than 3 blocks. The actual location of the block in the spectrum was not to be specified until after the auction. This was to ensure that a winning bidder received adjacent blocks of spectrum. Thus, bidders were facing a multiple unit auction with identical items, 12, in this case. Each block had a reserve price of DM 100 000 000. All blocks were sold simultaneously but each separately in its own ascending auction. Bidders could bid in as many of these separate auctions subject to eligibility requirements. The main features of the auction are summarized below:

1. Each bidder had to provide bank-guarantees, that limited the number of blocks (eligibility) and the highest bid they could place.
2. In the first round all bidders submit simultaneously their secret bids for the different blocks; they can submit as many bids as their eligibility permits;
3. Bids to be made in whole multiples of 100 000 DM;
4. For a bid in the first round to be deemed valid it had to exceed 100 million DM.
5. In subsequent rounds a bid for a block was valid if it exceeded the current highest bid by the current minimum bid increment. If there were no previous valid bid for this block, the minimum bid qualified as a valid bid.
6. The minimum increment, announced before the start of each round, was a percentage of the current highest bid on that block rounded up to the nearest 100 000 DM.
7. Newly submitted bids and standing highest bids from the previous round are deemed active.
8. The number of active bids of a bidder must not exceed the number of active bids in the previous round.
9. A bidder is eliminated from the auction, if he has at most one active bids in any round.
10. The auction ends when there are no valid bid submitted for any of the 12 blocks.

The first item prevents a winner reneging on his bid. Item (3) guards against signalling through the trailing digits of the bids. Items (4)–(5) ensure that prices ascend in the auction and do so at something other than a snail's pace. Items (8)–(10) encourage bidders to bid on two blocks and discourage them from 'sitting' on a block just to drive its price up. At the end, unsold blocks and additional blocks of auxillary spectrum were auctioned off in the second stage to bidders who had won a licence in the first stage.

To assess the design we need to know the goals of the auctioneer. The first question is why an auction? The alternative, administrative review (called beauty contests) requires government agents to divine the truth from those with an incentive to stretch it. This makes the selection open to charges of bribery, venality and favourtism. Auctions, certainly open ones, make cheating and bribery more difficult.[11] Second, they force the supplicants to support their claims for efficiency with real money instead of just talk. If one uses an auction should it be one that maximizes revenue? For a private entity perhaps but for a public one, not. In the case of the UMTS-auction, the government was primarily concerned with efficiency.[12]

9 A complete description may be found at `http://www.regtp.de/imperia/md/content/reg_tele/umts/9.pdf`.
10 Spectrum was paired so that potential users would have segment to send information and another to receive. While one portion of spectrum could do both this would require some kind of traffic control which is difficult to manage.
11 They also permit creative bidders to signal their intentions to others, so reducing competition.
12 See §11 of the German Telekommunikationsgesetz, `http://www.regtp.de/gesetze/start/in_04-01-00-00-00_m/index.html`

That is, does the auction allocate the licences to the ones that will make the most efficient use of them.[13] There is also the subsequent competition to provide communication services amongst the winners of the auction. To ensure sufficient competition, the government must ensure enough winners at the auction. Why an open auction? One reason is that it is difficult to accuse the auctioneer of favourtism. Another is the common values aspect of the spectrum auction alluded to above. Why a simultaneous ascending auction as opposed to a single auction where every unit was sold at once? After all, the units were identical in the eyes of the bidder. We suspect precedent.

There are other (open) auction forms one could have suggested. For example, allow bidders to submit simultaneous bids on packets of 2 or 3 blocks (at most one of which would be honored), for a possible mechanism along that line see Section "Analysis and an alternative" on page 37. Alas they have not yet been implemented.[14] Our guess is that the RegTP had no wish to be pioneers. The Simultaneous Ascending Auction (SAA) had been used in the US to much praise and so was a natural choice to consider. Given the choice of SAA, bundling the licences into blocks of two served to limit the exposure problem. If each licence had been auctioned off separately a bidder would have needed two of them to be able to provide service (most companies commented in the pre-auction-hearings that at least two – if not even three – blocks are necessary for providing high data-rates to customers). Limiting the number of blocks ensures sufficient competition in the delivery of services that consume the portion of spectrum being auctioned. There is still a threshold problem in that a bidder with bids on three blocks can crowd out two bidders with an interest in these three blocks. It is mitigated by the fact that bidders can switch between bidding for different blocks.

One other feature of the SAA that appealed to RegTM was that it would result in all blocks being sold for about the same price. This happens because bidders can always shift their subsequent bids to the block with the current lowest price. There is a heterogenous private values aspect to the auction in that incumbents, because of prior investments in technology and existing markets, assign a higher value to the licences than new entrants. This would suggest higher reserve prices for them (or subsidies for the entrants). The UMTS auction treated all bidders the same. It has been argued [3] that this, coupled with the 3 block limit it has been argued gave incumbent bidders an incentive to bid aggressivley not just to win but to prevent new entrants from winning.

What happened?

So much for theory. What happened? We describe the course the auction took by *phases*. A phase is constituted by a consecutive set of rounds, during which no bidder decreased his activity and the minimum increment was constant. Since dropping out of the auction requires reducing one's activity, the number of active bidders is constant during each phase. Our description is necessarily incomplete because RegTP provides for each round only two pieces of data. One is the highest bid and bidder in that round. The other item is a table that shows for each bidder the number of licenses he submitted the highest bids on.[15]

The auction began with seven bidders: Debitel Multimedia GmbH, DeTeMobil Deutsche Telekom Mobilnet GmbH, E-Plus Mobilfunk GmbH, Group 3 G, Mannesmann Mobilfunk GmbH, MobilCom Multimedia GmbH, and VIAG INTERKOM GmbH & Co. For the development of blockprices see Figure 1.

The first phase started with each bidder placing bids on three blocks. It continued with a minimum increment of 10 % in round 1 and ended between round 109 (the last time that Viag Interkom was the highest bidder on three blocks) and round 127 (Debitel dropped out). With few exceptions bids were raised only by roughly the necessary minimum increments. The major exception, MobilCom, put in bids, that were high enough to remain standing high bids for many rounds. On the first round they bid 1.002 BDM for a pair. This bid was topped for the first time in round 45. In round 46 MobilCom bid 1.602 BDM for two blocks; in round 61 they topped their own high bid with 2.5601 BDM; for an overview of the bidders' bidding behaviour see Figure 2.

The last rounds in which Group 3G, MobilCom and E-Plus were observed to be highest bidders on 3 blocks were 129, 136, and 136, respectively. So possibly during that time some additional phase-transitions happend. The next observable transition happend with round 139, since beginning with that round, the minimum increment precentage was reduced to 5 %. The minimum increment was reduced again during round 168, this time down to 2 %. The auction ran over fourteen days.

13 Public opinion, however, judges the success of an auction by the revenue generated.
14 However the US is about to do so. See announcement DA00-1486 (from `http://www.fcc.gov/wtb/auctions/700/da001486.pdf`) of the FCC to implement auction #31 as a combinatorial auction.
15 RegTP's counterpart in the US, the FCC, discloses all bids.

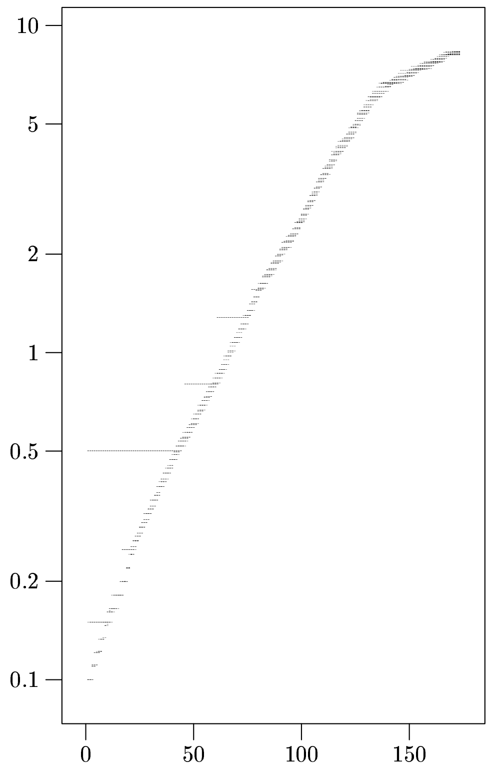

Figure 1. The abcissa depicts the round number and the ordinate depicts the blockprices in billion DM pn a logarithmic scale. Each highbid for a block is marked with a small horizontal line ( ).

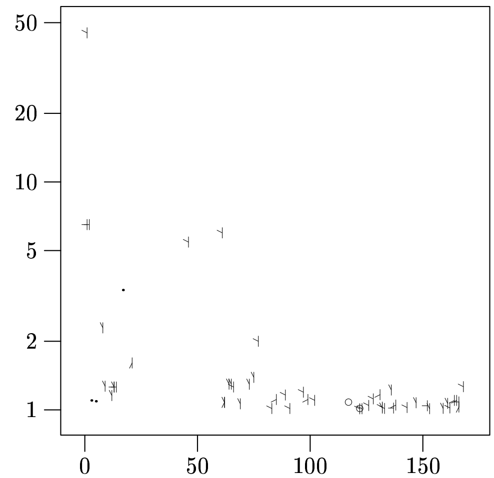

Figure 2. The abcissa depicts the round number and the ordinate depicts for each bidder and round the improvement he did by his new highbids (increment on that block divided by the product of previous high bid on that block and required minimumincrement on that block). Values less than 1.01 are omitted. T-Mobil (), MobilCom Multimedia (), Mannesmann Mobilfunk (), Group 3G (), E-Plus Hutchison(), VIAG Interkom(.), and debitel Multimedia (○).

The winners of the auction were DeTeMobil Deutsche Telekom Mobilnet GmbH, E-Plus Mobilfunk GmbH, Group 3 G, Mannesmann Mobilfunk GmbH, MobilCom Multimedia GmbH, and VIAG INTERKOM GmbH & Co. Each won two blocks at roughly 16 BDM. Total revenue raised by the auction's first stage was 98.8072 BDM.

Analysis and an alternative

Did the auction design promote efficiency? The answer depends on how bidders value the blocks. Suppose that values are independent and that the value of a third block is at most half the value of a pair of blocks, i. e., diminishing marginal benefits. In this case, a bidder should place bids on three licences as long as the total price of the three licences is less than the value he assigns to owning three licences. So in this case bidders can express their preferences and efficiency is promoted. If the value of the third licence exceeds half the value of a pair, then the exposure problem of the SAA makes bidders reluctant to place a bid for three blocks that exceeds 1.5 times the value for two blocks. This implies, that bidders can not express their preferences and this is an obstacle to efficiency.[16] We have assumed that the bidders are homogenous. However, there is good reason to believe that one group of firms (the incumbents) assigned a higher value to the licences than another group (the entrants). The incumbents in this auction were DeTeMobil Deutsche Telekom Mobilnet GmbH, E-Plus Mobilfunk GmbH, Mannesmann Mobilfunk GmbH, and VIAG INTERKOM GmbH & Co. These four firms had existing networks through which they offered second generation services. Thus the incremental cost of building a third generation network was smaller for them than the other three bidders. Also, a winner of the auction was permitted to offer second generation services. This made losing for incumbents more costly [4].

Given this heterogenity in bidders an auction in which all bidders face the same reserve price can never be revenue maximizing. Efficiency also dictates that when bidders are heterogenous, different bidders will usually pay different prices. This heterogenity in valuations has other implications. Since a winner of

16 Klemperer (2000) [p. 21] wonders why the incumbents did not settle for two blocks immediately after Debitel dropped out but first started going for three blocks and then, without any new information settle after many more billions of DM for two. He writes there: "Surprisingly, the dominant incumbents first pushed the prices up to almost UK levels, but then gave up and ended the auction before pushing any of the weaker firms – it is hard to construct beliefs about opponents for which this is rational behaviour." A possible explanation is that the dominant incumbents did not have decreasing marginal values and therefore worried about the exposure problem and hence refrained from bidding any higher for a triple and therefore did not push the opponents out.

two blocks has the right to offer second generation services, incumbent firms have an incentive to bid for three blocks so as to exclude an entrant. If this is the case, the auction limits competition in the provision of communication services. On the other hand, bidding for three licences opens the incumbent up to the exposure problem. Which of these two effects is dominant is difficult to see. As for an alternate mechanism, we propose an open multiround-auction, where in each round bidders can place a bid on obtaining two blocks and another bid on obtaining three blocks. Given these bids, it is for problems this small now computationally simple, to select a subset that maximizes value while using at most one bid from each bidder. Bids selected are called temporary winners.[17] A bidder is deemed active in any round, if he has a temporary winning bid in the previous round or if he has submitted a valid bid in the current round. If in any round, a bidder is active on exactly two (three) blocks, his eligibility number is set at two (three). If his eligibility number drops below 2, he's cast out. But his bids remain in the system. A bidder with an eligibility number of three can place bids for triples and pairs. A bidder with an eligibility number of two can only place bids for pairs. To keep the auction moving one must specify a minimum increment. When bids on differente combinations are allowed, how to define that minimum increment becomes problematic. Bids on triples may require different increments than bids on pairs. This may be necssary to mitigate the threshold problem.

Our suggestion for the minimum increment on pair bids is described below. It requires the auctioneer to specify three numbers x, y and z. The minimum increment for a bid on a pair is the minimum of two numbers described below, but at least a small percentage z of the smallest per item value of the temporary winners.

1. The difference between $100 + x\,\%$ of the lowest winning pair if there is any or otherwise $+\infty$ and the bidder's current bid for a pair.

2. The difference between the sum of the two lowest temporary triple-winners minus this bidders current pair bid minus up to two highest pairbids that were temporary losers from other bidders. Take $y\,\%$ of one third of this amount. (Here y could be a number between 0 and 100 but a value around 20 is perhaps sensible.)

A bidder's pair bid is valid, if it is greater than or equal to his last pair bid plus the corresponding minimum increment and his eligibility is at least two. One can define a similar increment for bids on triples which we omit for space considerations. The first of these numbers x follows the usual notion of an increment above some previously winning bid. The second number y is set to reduce the threshold problem. The third number z prevents the increment from declining and prolonging the auction.

This scheme has four advantages over the one used in the UMTS-auction. First by permitting combinatorial bids, it allows the bidders to express valuations with increasing marginal values. For example, a triple is worth more than 1.5 times the value of a pair. Second, when the demand for blocks equals their supply it avoids the coordination problem of which firm gets which block. Third, economic efficiency is improved. Fourth, the auctioneer can benefit by obtaining a higher revenue.

Conclusion

The theory of auctions of multiple objects is far from complete with many mathematical obstacles. We have sought only to give a flavor of the analysis and its relevance to the design of auctions.

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17 In the absence of data, on could copy the reserve prices, by setting them at 200 MDM for a pair and 300 MDM for a triple.