COGA

Combinatorial Optimization & Graph Algorithms

Locomotives and Rail Cars: Switching, Routing, and Scheduling

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Practical Background

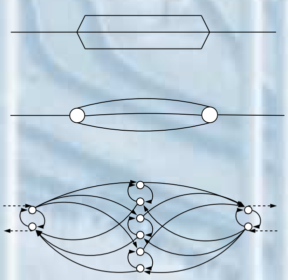
Industrial switching involves moving materials on rail cars within or between industrial complexes and connecting with other rail carriers. A *transportation request* specifies a set of rail cars to be relocated from their current origin(s) to destination(s), possibly within a deadline. Two planning tasks arise; in the *low level switching* all cars from a request have to be collected and grouped on a suitable track. This requires to give a highly detailed sequence of tasks like coupling and decoupling, switching of cars, intermediate parking of cars, and the precise making up of the train. The savings potential of this first step is not in terms of time but rather in the simultaneous making up of several trains in one sequence of switching operations.

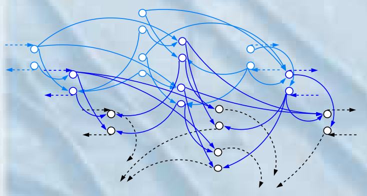
In the subsequent *routing and scheduling* stage one has to assign a locomotive to each train, and find a path to its target track(s). Such a path must obey several feasibility constraints like the locomotive's mode of operation (pushing or pulling from front or rear makes a difference, and a proper switching between modes has to be ensured); temporarily blocked tracks have to be avoided; the length of the train may prohibit stopping at certain tracks; the train's deadline must be respected; other trains may be picked up en route; and most importantly, a path must not cross other paths (in space-time). That is, on its way, a locomotive blocks a collection of tracks for a certain time, and the routing and scheduling of all locomotives has to be done simultaneously.

A human planner must often resort to routine and simple heuristics, not least for the reason of unavailability of computer aided suggestions. A short term practical goal in our project is to come up with a *feasible* schedule, which cannot be guaranteed by the current planning practice. In the long run, our proposals should lead to a reduction of the number of necessary locomotives.

Integer Program

The low level switching stage is an interesting and challenging combinatorial optimization problem in itself. Here, we only present a formulation of the routing and scheduling stage. The graph representing the track infrastructure is enhanced in order to control the direction from which a track is entered and left (and some more operational details), resulting in the so-called *switching operations graph* which is then time expanded.





A path $p \in P_k$ in this network encodes a feasible route and schedule for locomotive $k \in K$. The following integer program is almost self suggesting. A coefficient d_i^p is 1 iff request i is served in schedule p. A coefficient $a_{b,t}^p$ is 1 iff track b is used at time t in schedule p. The binary variable x_k^p takes value 1 iff the schedule p is assigned to locomotive k. The constraints ensure that each train is routed exactly once and no track is used by two locomotives at the same time instant.

$$
\begin{aligned}
\min \quad & \sum_{k\in K} \sum_{p\in P_k} c_k^p x_k^p & & \\
\text{s.t.} \quad & \sum_{k\in K} \sum_{p\in P_k} d_i^p x_k^p = 1 & & \forall \text{ request } i \\
& \sum_{k\in K} \sum_{p\in P_k} a_{b,t}^p x_k^p \le 1 & & \forall \text{ track } b, \text{ at time } t \\
& \sum_{p\in P_k} x_k^p \le 1 & & \forall \text{ locomotive } k \\
& x_k^p \in \{0,1\} & & .
\end{aligned}
$$

Of course, this seemingly compact model hides its complexity in the innocent looking "$p \in P_k$." The exponential number of path variables calls for solving the linear programming relaxation by column generation.

Branch & Price

This popular technique has almost attained an industry standard level and usually enables us to set up appropriate models relatively quickly. Even though recent years brought us rather generic frameworks, the actual implementation of a branch & price code is still non-trivial and sometimes needs heavy tailoring to the particular application. Our pricing problem is a resource constrained shortest path problem in the time expanded shunting operations graph and allows us to incorporate a wealth of practical side constraints.

Timeline

The proposed schedule of this recently launched research project is quite tough. Within half a year we would like to run first real-time real-life tests with a prototype implementation which will be gradually extended according to the feedback we get. Our algorithms are planned to be installed as part of an existing software at a German steel works' in-plant railroad.

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