Theory of Krylov subspace methods
Lecture in the Summer Semester 2012
Prof. Jörg Liesen (13.03.2012)

Time and Room: Wednesday, 14-16, MA 143

LP: 5 (2 SWS)

Language: English

The module description (see my homepage: www.math.tu-berlin.de/?78578) gives some details on the content of the lecture and background literature. In addition, the following description gives an overview of the plan for this semester:

Krylov subspace methods can be described and derived from different points of view. In this lecture we will focus on the idea of *moments*. We will start with (a simplified version of) Stieltjes' classical moment problem and its solution via mechanical quadrature methods. This will lead us in particular to the Gauss-Christoffel quadrature of the Riemann-Stieltjes integral, which is given through Stieltjes' moment problem. We will study properties of the orthogonal polynomials underlying the Gauss-Christoffel quadrature (among them the three-term Stieltjes recurrence, interlacing of roots, formulation in terms of continued fractions). The matrix analogoues of these results will be found by using Jacobi matrices, which will bring us to the Lanczos algorithm and the conjugate gradient method. Vorobyev's moment problem and its generalisations will lead to more general algorithms in the Krylov subspace context, and here particularly the Arnoldi algorithm and the GMRES method.

All in all, the lecture will give an alternative and rarely presented approach to Krylov subspace methods, which is helpful to obtain a better understanding of this important class of iterative methods. The results presented in the lecture can be of interest to anyone working in numerical mathematics and particularly on the numerical solution of linear algebraic problems such as linear algebraic systems, eigenvalue problems, or least squares problems.

Prerequisites: It is clear from the above, that the lecture will address numerous topics in the areas of linear algebra, analysis and numerical mathematics. The lecture is intended for *advanved* Bachelor students as well as Master and PhD students. Required prerequisites are: Lineare Algebra I+II, Analysis I+II, Numerische Mathematik I (possibly II) and Numerische Lineare Algebra I.