# Berlin-Zürich summer school 2009 in Chorin

## Titles and abstracts of the contributed talks by participants

# Local degree distribution in scale free random graphs

**Agnes Backhausz (Eötovös University, Budapest)**

# Backward SDEs with superquadratic growth

**Xiaobo Bao (ETH Zürich)**

We discuss the solvability of backward stochastic differential equations (BSDEs) with superquadratic generators. We first prove that given a superquadratic generator, there exists a bounded terminal value, such that the associated BSDE does not admit any bounded solution. On the other hand, we prove that if the superquadratic BSDE admits a bounded solution, then there exist infinitely many bounded solutions for this BSDE. Finally, we prove the existence of a solution for Markovian BSDEs where the terminal value is a bounded continuous function of a forward stochastic differential equation.

# Predictable projections of Itô SDEs

**Matteo Casserini (ETH Zürich)**

By examining some properties of the predictable projection of complex stochastic processes, we obtain interesting results in connection with the representation of It SDEs as projection of corresponding complex cases. This approach seems to be unexplored and has some remarkable consequences in connection with numerical methods, especially with the cubature method on Wiener spaces (developed by Lyons and Victoir in 2003).

# Quantum spin systems: a dynamical approach

**Alessandra Cipriani (Universtität Zürich)**

In this work we study some properties of quantum spin systems: in particular we consider Gibbs fields on a lattice obtained from a classical Hamiltonian by adding a *transverse field*. Via a stochastic-geometric representation we are able to reformulate means and correlations of the quantum model as *classical* means and correlations with respect to a random field on "loops". We the define a

Markov process which is reversible with respect to the distribution of this field and via the Bochner-Bakry-Emery method (or $\Gamma_2$ method) we obtain explicit high temperature conditions for the positivity of the spectral gap and finally a stretched exponential decay of correlations.

# Ballisticity criteria for RWRE

**Alex Drewitz (TU Berlin)**

Consider a random walk in a uniformly elliptic i.i.d. random environment in dimensions $d \geq 2$. In 2002, Sznitman introduced for each $\gamma \in (0,1)$ certain conditions $(T)_\gamma$ as well as $(T')$, the latter being defined as the fulfilment of $(T)_\gamma$ for all $\gamma \in (0,1)$. He proved that $(T')$ implies ballisticity and that for each $\gamma \in (0.5,1)$, $(T)_\gamma$ is equivalent to $(T')$. It is conjectured that this equivalence holds for all $\gamma \in (0,1)$. Here we prove that for $\gamma \in (\gamma_d,1)$, where $\gamma_d$ is a dimension dependent constant taking values in the interval $(0.366, 0.388)$, $(T)_\gamma$ is equivalent to $(T')$.

# Maximum Likelihood Estimation for Lévy-driven Ornstein-Uhlenbeck processes

**Hilmar Mai (HU Berlin)**

We develop a maximum likelihood approach for estimating the coefficient of a Lévy driven Ornstein-Uhlenbeck process. In order to do this we prove that the laws of the Ornstein-Uhlenbeck processes corresponding to different coefficients are mutually absolutely continuous if and only if the background driving Lévy process has a diffusive component. Then, we give conditions such that the maximum likelihood estimator exists uniquely and is strongly consistent as well as asymptotically normal. To obtain these results we show that the class of Ornstein-Uhlenbeck processes corresponding to different coefficients forms a curved exponential family of stochastic processes. Finally, we investigate a discretized version of our estimator and discuss a simulation study to asses the finite sample behavior and demonstrate its practical tractability.

# Multitype branching processes conditioned on very late extinction. An example in epidemiology

**Sophie Pénisson (Universti?t Potsdam)**

We consider for a multitype continuous-time branching process the conditioned distribution $\mathbb{P}(X_t \in \ldots . | X_{t+\theta} > 0, \lim_{s\to\infty} X_s = 0)$, for some $\theta \geqslant 0$. We investigate both limits as $\theta$ and $t$ tend to $\infty$, as well as the commutativity between them. The first limit stands for a conditioning on very late extinction, while the second limit is a generalization of the Yaglom distribution (obtained for $\theta = 0$). We shall investigate the same limits for a multitype Feller diffusion process, as well as the connexion between the conditioned Feller and branching processes.

In terms of risk analysis, the study of the conditioned distribution $\mathbb{P}(X_t \in . | X_{t+\theta} > 0)$ for $\theta$ very large is of some interest. If the process represents the size of an infected population, this distribution corresponds indeed to the worst scenarios of the epidemic. We shall apply some of the above-mentioned results to the modelling of the propagation of BSE, and estimate the contamination parameter of this model in this setting.

## Pólya's Urn and its Perspective on Point Processes

**Mathias Rafler (Universität Potsdam)**

The aim of this talk is to generalise the construction of the Poisson process to obtain the so-called Pólya sum process. This process is inspired by the dynamics of Pólya's urn. We explore its basic properties and highlight parallels as well as differences to the Poisson process. The Pólya sum process is a primer example of a rich class of point processes, which are characterised by partial integration formulae.

## Erdős-Rényi random graphs + forest fires = Self-Organized Criticality

**Balázs Ráth (Budapest University of Technology)**

## Differentiability of martingale driven Backward SDE and application to finance

**Anja Richter (HU Berlin)**

In this talk we consider quadratic growth BSDE driven by continuous local martingales. We first derive the Markov property of a forward-backward system when the driving martingale is a strong Markov process. Then we establish the differentiability of a FBSDE with respect to the initial value of its forward component. It enables us to obtain the main result of this talk that is to describe the control process of the BSDE in terms of a differential operator of the solution process and the correlation coefficient of the forward process. This formula generalizes the results obtained by several authors in the Brownian setting, designed to represent the optimal delta hedge in the context of cross hedging insurance derivatives that generalizes the derivative hedge in the Black-Scholes model. It involves Malliavins calculus which is not available in the general martingale setting. Consequently, we propose a new method based on stochastic calculus techniques. This is a joint work with Peter Imkeller and Anthony Reveillac.

## Scaled limit and rate of convergence for the largest eigenvalue from the generalized Cauchy random matrix ensemble

**Felix Rubin (Universität Zürich)**

In this talk, we are interested in the asymptotic properties for the largest eigenvalue of the Hermitian random matrix ensemble, called the Generalized Cauchy ensemble $GCy$, whose eigenvalues PDF is given by

$$\text{const} \cdot \prod_{1 \le j < k \le N} (x_j - x_k)^2 \prod_{j=1}^{N} (1 + i x_j)^{-s-N} (1 - i x_j)^{-\overline{s}-N} dx_j,$$

where $s$ is a complex number such that $\Re(s) > -1/2$ and where $N$ is the size of the matrix ensemble. We will see that for this ensemble, the appropriately rescaled largest eigenvalue converges in law. We also express the limiting probability distribution in terms of some non-linear second order differential equation. Eventually, we show that the convergence of the probability distribution function of the re-scaled largest eigenvalue to the limiting one is at least of order $(1/N)$.

## Exact asymptotics in the parabolic Anderson model

**Adrian Schnitzler (TU Berlin)**

In this talk we consider annealed time asymptotics for the parabolic Anderson model, i.e. the Cauchy problem for the heat equation on the lattice with random potential, with homogeneous initial value:

$$\begin{cases} \frac{\partial}{\partial t} u(t,x) = \kappa \Delta u(t,x) + \xi(\omega,x) u(t,x), & (\omega,t,x) \in \Omega \times \mathbb{R}_+ \times \mathbb{Z}^d, \\ u(0,x) = \mathbb{1}, & x \in \mathbb{Z}^d. \end{cases}$$

We show how to derive exact asymptotics for expressions such as

$$\left\langle \prod_{i=1}^{p} f_i \left( u \left( t_i(t), 0 \right) \right) \right\rangle,$$

where $\langle \cdot \rangle$ denotes expectation with respect to the potential.
We consider the case of an i.i.d. field of nonnegative random variables with tails that decay faster than those of a double exponential distribution. This can be applied to investigate intermittency and ageing properties of the model.

## Random walk trajectories and random interlacements

**Augusto Teixeira (ETH Zürich)**

The model of random interlacements was recently introduced by A-S Sznitman as a natural tool to analyze random walk trajectories on large graphs. Intuitively, this models describes the "local picture" left by the random walk when it runs up to a certain time. In this talk we intend to describe the link between these two models, with emphasis on their percolative properties.

## On a generalization of the Sherrington-Kirkpatrick model

**Philipp Thomann (Universität Zürich)**

Many calculations on the SK-Model use its quite special properties. In order to understand the problem more thoroughly we treat a generalized version where spins are not constrained to Ising type. We give a proof of our previous conjecture on the free energy by using the ideas appearing in Talagrand (2009).

## Skorohod-reflection of Brownian paths and $BES^3$

**Bálint Vető (Budapest University of Technology)**

## Maximum of a rough path from its signature

**Danyu Yang (Oxford University)**

We try to extract the maximum value of a rough path from its signature. Since different norms could be used on the tensor algebra, we concentrate on the maximum of coordinate iterated integrals. After reparametrization, the coordinate iterated integrals of process X are Hlder continuous functions. Furthermore, polynomial of coordinate iterated integral of X is a linear combination of higher ordered coordinate iterated integrals of X. Thus, we will try to extract the maximum of f from a series of polynomials of f.