Workshop on Duality of Markov Processes

Abstracts

6. and 7. November 2014

Mareike Esser and Sebastian Probst: Duality between the Moran Model with recombination and a marginalised ARG

Abstract: Due to the huge state space, it is notoriously difficult to find the full distribution of types in a sample that undergoes recombination. If in addition the recombination rate is large, the usual diffusion limit approach does not apply and computational methods become less tractable. We therefore take an alternative route by concentrating on joint probabilities of types at tuples of loci. This leads to a formal duality relation between the type distribution process of the Moran Model with recombination forward in time and a marginalised version of the Ancestral Recombination Graph backward in time. Besides, the corresponding duality function yields us an insight on the interplay of drawing with and without replacement.

Cristian Giardinà: Stochastic dualities and Lie algebras

Abstract: An algebraic approach to duality theory of Markov processes will be described. In particular, the scheme will be explained using classical Lie algebras for some well-know processes in population dynamics (such as multi-type Wright-Fisher diffusion with mutations), as well as for some new processes that arise by considering deformed algebras.

Adrián González Casanova and Maite Wilke Berenguer: The seed-bank coalescent

Abstract: We present a new natural coalescent model for populations under the influence of seed-banks. In particular, we derive the scaling limit of a Wright-Fisher model with geometric seed-bank component, characterize its dual, identify its genealogical structure and define a new canonical coalescent model which we call seed-bank coalescent. Finally we discuss some of its properties, for example the fact that it does not come down from infinity. This is a joint work together with Jochen Blath and Noemi Kurt.

Andreas Greven: Duality in population models: from configurations to genealogies

Abstract: We discuss duality techniques for spatial Fleming-Viot and Moran models with selection and mutation, both on the level of measure-valued processes (type-frequences at sites) and for the genealogies of the population modelled as marked ultrametric measure spaces.

Felix Hermann: Using duality in the partial duplication random graph

Abstract: To all classical examples where dual Markov processes are used, we add a piecewise deterministic jump process which is dual to the evolution of the characteristic function of the degree distribution of the (continuous-time version of the) partial duplication random graph. Starting in a fixed graph, once a vertex is added, an existing vertex is picked and each of its connecting edges are copied with probability p, which then form the neighbors of the new vertex. Using our duality, we are able to show that there is a large connected component iff $p > p^*$, where $p^* \approx 0.57$ is the unique solution of $p \cdot e^p = 1$. This is joint work with Peter Pfaffelhuber.

Sabine Jansen: Duality of Markov processes with respect to a function

Abstract: Despite its usefulness in diverse areas from queuing theory to mathematical population genetics, there is no general theory of duality of Markov processes with respect to a function. This is in stark contrast to duality with respect to a measure, which has been well-studied in the context of potential theory. This talk presents a systematic study of the notion of duality with respect to a function; the focus is on elucidating the analytic framework, building on notions of dual pairs in functional analysis and convex geometry. In addition, we address questions such as why some dualities are associated with stochastic monotonicity. The talk is based on joint work with Noemi Kurt (Probab. Surveys 11, 59-120 (2014)).

Tom Kurtz: Filtering and the existence of intertwining relations

Abstract: The general filtering problem for a Markov process is concerned with identifying the conditional distribution of the process state given partial information about the process. Martingale characterizations of these conditional distributions have as a corollary a general result on the existence of intertwining relations. The basic characterization will be given and a variety of examples discussed.

Ute Lenz: A Siegmund dual for the line counting in a pruned ancestral selection graph

Abstract: In a (two-type) Wright-Fisher diffusion with directional selection and two-way mutation, the pruned lookdown ancestral selection graph (pruned LD-ASG) describes the evolution of a minimal set of potential ancestors of one individual sampled at some time in the far future. It is a backward in time construction that contains elements of both the ancestral selection graph and the lookdown construction and includes pruning of certain lines upon mutation. We develop a forward in time process, which turns out to be a Siegmund dual for the line counting process of the pruned LD-ASG. From a first step analysis of this dual process one can read off a recursion that determines the tail probabilities of the equilibrium distribution of the number of lines in the pruned LD-ASG. This is joint work with Ellen Baake and Anton Wakolbinger.

Martin Möhle: Dualities, cones and spectral decompositions arising in mathematical population genetics

Abstract: It is shown that cone duality essentially coincides with Liggett's definition of duality of Markov processes. Several examples, mainly motivated from mathematical population genetics, of dual Markov processes and their corresponding convex cones are provided, including Fleming-Viot measure valued processes and their dual exchangeable coalescents allowing for simultaneous multiple collisions of ancestral lineages.
For haploid models duality usually turns the forward transition mechanism into a triangular backward transition mechanism. We go one step beyond and provide some diagonalization results/spectral decompositions for exchangeable coalescents with an emphasis on the Bolthausen-Sznitman coalescent.

Peter Seidel: A dual process for the spatial Moran model with mutation and selection carrying the family decomposition with founding fathers

Abstract: The spatial Moran model is a locally finite and multi-type population model in which each individual of the population inhabits a site in geographic space and has one of a finite number of different genetic types. Its evolution is given by migration, general type-dependent mutation, resampling and selection.
Our general goal is to model the evolution of genealogical information forward in time, in particular, the evolution of ancestral lines and genealogical distances. For this talk we assume that both the population and the set of sites are finite and consider the following reduced genealogical information. At time t we assign to each life-site representing an individual currently alive besides its site and type also its ancestor alive at time 0 which is a member of the first generation of individuals to which we refer

as founding fathers, that is, identifying the set of life-sites with the same founding father as a family we obtain a family decomposition of the current population which is enriched by the information on the founding fathers.
As a result we introduce a special new dual process determining the law of both the site-type information and the family decomposition with founding fathers of a tagged subset of life-sites for any fixed time t by a Feynman-Kac duality. In addition, we discuss the main features of this dual process and give some applications of the Feynman-Kac duality.

Anja Sturm: On extensions of Gray's and Siegmund's dualities and applications

Abstract: Siegmund proved that (almost) every monotone Markov process taking values in a totally ordered set has a dual, that is also a monotone Markov process, taking values in (almost) the same state space. Later, Gray introduced a general duality for monotone spin systems, which are taking values in a partially ordered set. In general, this dual process takes values in a much larger state space. Nonetheless, Gray showed how certain well-known dualities, such as for the contact and the voter model, fit into his framework.
In this talk, we construct a dual for monotone Markov processes taking values in a general partially ordered set, obtaining Gray's and Siegmund's duals as special cases. We also derive a condition under which the state space of the dual simplifies greatly. Even if this condition is not satisfied, the resulting rather complicated dual process may sometimes be simplified if one is satisfied with a subdual. Examples and applications include cooperative branching systems, for which a subduality is used.
This is joint work with Jan Swart (UTIA Prague).

Jan Swart: Some dualities for a class of one-dimensional cancellative systems

Abstract: For one-dimensional particle systems that treat the two spin values symmetrically, the set of lattice points where one spin borders the other is itself a Markov process, called the interface model. For cancellative systems, I will show how this observation can be combined with the usual cancellative systems duality to obtain some unusual duality relations, that can be used to prove that if one type cannot invade the other (in a sufficiently strong way), then the process clusters.