13. Practice sheet for the lecture: **Felsner/ Kleist**
Combinatorics (DS I) 12. July 2017

Due dates: 18.-20. July
`http://www.math.tu-berlin.de/~felsner/Lehre/dsI17.html`

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(1) Let $D = (\{v_1, \ldots, v_n, s, t\}, \vec{E})$ be a directed graph. We are interested in the number of Hamilton paths from s to t. Develop a formula which can be evaluated in $o(n!)$. [Hint: Consider the walks of length $n+1$ from s to t and use Möbius inversion.]

(2) In the lecture we saw a lower bound on the number of perfect matchings of 3-regular bipartite multi-graphs with $2n$ vertices. Show that this lower bound is essentially best-possible (with the following steps):

   (a) Let $\mathcal{G}$ be the set of all 3-regular bipartite multi-graphs, where vertices and edges are labelled. How many are there?

   (b) Let M be a set of n labelled edges. Count the number of graphs $G \in \mathcal{G}$ which contain M as a perfect matching.

   (c) Conclude that the bound is best-possible.

(3) Let F_n denote the nth Fibonacci number. Use Kasteleyn signatures to find matrices $M(n)$ with

$$\det(M(n)) = F_n.$$

[Hint: Recall the model of 1-2-sums for the Fibonacci numbers.]