
**7. Practice sheet for the lecture:
Combinatorics (DS I)**

Felsner/ Kleist
31. May 2017

Due dates: 06.-08. June

<http://www.math.tu-berlin.de/~felsner/Lehre/dsI17.html>

- (1) Let $P := (M, \leq)$ be a finite poset with n elements, i.e. $|M| = n$. For a given ordering of the elements (m_1, \dots, m_n) , we can associate with P a relation matrix $(a_{ij}) = A \in \{0, 1\}^{n \times n}$, such that $a_{ij} = 1 \Leftrightarrow m_i \leq m_j$.
- (a) Show that it is possible to find an ordering on (m_1, \dots, m_n) such that A is an upper triangle matrix?
- (b) Show that $\#\{(i, j) \mid a_{ij} = 1\} = \binom{n+1}{2} \Leftrightarrow P$ is a total order.
- (c) Find matrix properties for a $\{0, 1\}$ -matrix B , which imply that B represents a partial order relation?
- (2) A poset $P = (X, \leq)$ is *ranked* if there exist a function $r : X \rightarrow \mathbb{N}$ such that for all $x \in X$, the rank $r(x)$ is the length of *all* maximal chains ending in x .
For $n \in \mathbb{N}$, the *divisor-poset* P_n is the set of all divisors of n ordered by divisibility:

$$P_n := \{x \in \mathbb{N} : x \mid n\}, \{(x, y) \in \mathbb{N}^2 : x \mid y \text{ and } y \mid n\}.$$

Prove that P_n is ranked. Sketch the Hasse-Diagramm of P_{60} and compute its ranks. Is P_n a lattice?

- (3) q -analogues
- (a) How many different maximal chains of nested subspaces of $V_n(q)$ exist?
- (b) Show the q -analogue of the LYM inequality:

$$\sum_{k=0}^n \frac{p_k(\mathcal{A})}{\begin{bmatrix} n \\ k \end{bmatrix}} \leq 1$$

- (4) Let $k \in \mathbb{N}$ be fix. Prove that for each $n \in \mathbb{N}$ there are unique $a_k > a_{k-1} > \dots > a_t \geq t \geq 1$ with $a_i \in \mathbb{N}$, such that

$$n = \binom{a_k}{k} + \binom{a_{k-1}}{k-1} + \dots + \binom{a_t}{t}.$$

- (5) Show that every finite poset is isomorphic to the containment order on some family of sets.
- (0.1) We will discuss exercise 1 of the 0. sheet.