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Robert Juričević, University of Waterloo

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Abstract

Let α and β be roots of the quadratic polynomial $X^2 - \sqrt{L}X + M$, where $L = (\alpha+\beta)^2 \in \mathbb{Z}$, $M = \alpha\beta \in \mathbb{Z}$, $\gcd(L, M) = 1$, and $D = L - 4M = (\alpha - \beta)^2$. We define a Lehmer sequence $(u_n)_{n=0}^{\infty}$ and companion Lehmer sequence $(v_n)_{n=0}^{\infty}$ associated with the pair (α, β), labelled a Lehmer pair, by

$$u_n = \frac{\alpha^n - \beta^n}{\alpha^\delta - \beta^\delta}, \;\; v_n = \alpha^n + \beta^n,$$

where $\delta = 1$ or 2 if n is odd or even respectively. A Lucas sequence corresponds to a Lehmer sequence for $\sqrt{L} \in \mathbb{Z}$ with associated Lucas pair (α, β). Classically, note that the Fibonacci sequence

$$0, 1, 1, 2, 3, 8, 11, 19...$$

corresponds to a Lucas-Lehmer sequence for $(\sqrt{L}, M) = (1, -1)$, with associated real Lucas pair $(\alpha, \beta) = ((1+\sqrt{5})/2, (1-\sqrt{5})/2)$, while the Mersenne sequence

$$3, 7, 31, 127, 8191, 131071, 524287, 214783647, \ldots$$

corresponds to a subsequence of a Lucas-Lehmer sequence for $(\sqrt{L}, M) = (3, 2)$, with associated integer Lucas pair $(\alpha, \beta) = (2, 1)$. On the other hand, we say that a prime number p is a primitive divisor of a term u_n of the Lehmer sequence if $p|u_n$, but $p \nmid LDu_3 \cdots u_{n-1}$. Note that $LD = (\alpha^2 - \beta^2)^2$. Recently, in a spectacular display of the interplay between computational number theory and theoretical number theory in helping to resolve an outstanding problem, Bilu, Hanrot, Voutier, and Mignotte (J. Reine Angew, 2001) classified all triples (n, α, β) such that (α, β) is a Lehmer pair, and u_n has no primitive divisor. We will discuss our recent computational work on primitive divisors of Lehmer sequences. Moreover, we will exhibit a connection between these sequences and the arithmetic of squares and cubes.

Robert Juricevic, Department of Pure Mathematics, University of Waterloo, Waterloo, Ontario, N2L 3G1, CANADA
E–mail: rjuricevic@math.uwaterloo.ca