DISCRETE GEOMETRY I

1. Week: Convex sets, convex hulls, Minkowski sums, Carathéodory's theorem.
   Main source: [Bar02, Chapter I, §1 and §2]
   Supplementary sources: [Mat02, §1.2], [Grü03, §2.1 and §2.3], [JT08, §2.3], [Zie95, §1.6]
2. Week: Radon's theorem, Helly's theorem, Euler characteristic.
   Main source: [Bar02, Chapter I, §4 and §7]
   Supplementary sources: [Mat02, §1.3 and §1.4], [Grü03, §2.3]
3. Week: Fourier–Motzkin elimination, Affine sub- and halfspaces, isolation theorem, dimension, faces, supporting hyperplanes.
   Main source: [Bar02, Chapter I, §9 (only 9.1 and 9.2) and Chapter II, §1 and §2]
   Supplementary sources: [Mat02, §1.1], [Grü03, §2.4], [JT08, §2.3], [Zie95, §1.2]
4. Week: Extreme points, Krein–Milman theorem, polyhedra, convex cones, conic hull.
   Main source: [Bar02, Chapter II, §3, §4 and §8]
   Supplementary sources: [Grü03, §2.4]
5. Week: Closed convex sets, separation theorems, polar sets, bipolarity, polytopes.
   Main source: [Bar02, Chapter II, §16 and Chapter III §1, Chapter IV §1 (only up to 14)]
   Supplementary sources: [Mat02, §1.2], [Grü03, §2.2], [JT08, App. B]

Next we take a closer look at polytopes and in particular their combinatorics. Our main sources are [Bar02] and [Grü03].

6. Week: Faces and polars of polytopes, f-vector, Euler–Poincaré formula.
   Main source: [Bar02, Chapter VI, §1 and §3]
   Supplementary sources: [Grü03, §3.1, §3.4], [JT08, §3.2, §3.3], [Mat02, §5.1—5.3], [Zie95, §2.1—2.3]
7. Week: Combinatorial equivalence, Schlegel diagrams, examples.
   Main source: [Grü03, §3.2, §3.3, §4.1—4.4]
   Supplementary sources: [Zie95, §0, §2.2, §5]
8. Week: Birkhoff polytope, permutation polytope, Schur–Horn theorem, transportation polytope.
   Main source: [Bar02, Chapter II, §5—7]
9. Week: Cyclic and simple polytopes.
   Main source: [Bar02, Chapter VI, §4 and §5]
   Supplementary sources: [Grü03, §4.5, §4.7], [Mat02, §5.4], [Zie95, §2.5]
10. Week: h-vector, Dehn–Sommerville equations, upper bound theorem.
    Main source: [Bar02, Chapter VI, §6 and §7]
    Supplementary sources: [JT08, §3.5], [Mat02, §5.5]
11. Week: Inductive construction of polytopes, d-polytopes with $d+2$ vertices, f-vectors of 3-polytopes.
    Main source: [Grü03, §5.2, §6.1, §10.3]
    Supplementary sources: [Zie95, §4, §6.5]

Finally we will consider polytopes under an *algorithmic* point of view. Our main source is now the book by Joswig and Theobald [JT08].

12. Week: Linear programming, simplex method.
Main source: [JT08, §4.1—4.4]
Supplementary sources: [Bar02, Chapter IV, §6—8], [Sch86, §7.4, §7.5, §7.9, §11], [Zie95, §3.2]
13. Week: Computation of convex hulls, double description method.
Main source: [JT08, §5.1—5.3]

References

[Bar02] Alexander Barvinok. *A course in convexity*, volume 54 of *Graduate Studies in Mathematics*. American Mathematical Society, Providence, RI, 2002.

[Grü03] Branko Grünbaum. *Convex polytopes*, volume 221 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 2003. Prepared and with a preface by Volker Kaibel, Victor Klee and Günter M. Ziegler.

[JT08] Michael Joswig and Thorsten Theobald. *Algorithmische Geometrie*. Vieweg Studium: Aufbaukurs Mathematik. [Vieweg Studies: Mathematics Course]. Vieweg, Wiesbaden, 2008. Polyedrische und algebraische Methoden. [Polyhedral and algebraic methods].

[Mat02] Jiří Matoušek. *Lectures on discrete geometry*, volume 212 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2002.

[Sch86] Alexander Schrijver. *Theory of linear and integer programming*. Wiley-Interscience Series in Discrete Mathematics. John Wiley & Sons, Ltd., Chichester, 1986. A Wiley-Interscience Publication.

[Zie95] Günter M. Ziegler. *Lectures on polytopes*, volume 152 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1995.