ALGORITHMIC DISCRETE MATHEMATICS III:  
EXERCISES 1  

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The convex hull of finitely many points with 0/1-coordinates is a 0/1-polytope.

**Exercise 1.** Give bounds for the vertex and facet complexities of the 0/1-polytopes in dimension $n$.

Two polytopes are *combinatorially isomorphic* if their face posets are isomorphic (as posets).

**Exercise 2.** Enumerate the 3-dimensional 0/1-polytopes up to combinatorial automorphisms. Can you make polymake visualize them all in one picture?

**Exercise 3.** Let $P$ and $Q$ be convex polytope. Describe an algorithm for deciding whether $P$ and $Q$ are combinatorially isomorphic. Do some examples with polymake.

A *Schlegel diagram* of a polytope is a central projection onto one of its facets, say $F$, yielding a polyhedral subdivision of $F$ by coning with the center of the projection over all facets other than $F$ (and intersecting with $F$).

**Exercise 4.** Construct a 4-dimensional polytope which is not combinatorially isomorphic to any 0/1-polytope. Let polymake draw a Schlegel diagram. Generalize this to arbitrary dimension.

A polyhedron is *pointed* if it does not contain any affine subspace of positive dimension.

**Exercise 5.** Let $P \subset \mathbb{R}^n$ be a polyhedron. Show that $P$ is pointed if and only if there is an affine transformation $T$ such that $T \cdot P$ is contained in the nonnegative orthant $\mathbb{R}_+^n$. What can you say about projective transformations?

**Exercise 6** (Recap). Phase II of the simplex method for linear programming requires to start with one known vertex of the feasible region. Describe a method for phase I, i.e., an algorithm which takes $A \in \mathbb{Q}^{m \times n}$ and $b \in \mathbb{Q}^m$ as input and produces one vertex of $P(A, b) = \{ x \in \mathbb{Q}^n \mid Ax \leq b \}$. You may assume that $P(A, b)$ is pointed.

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