polymake for integer linear programming
ISMP 2012

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w/ Ewgenij Gawrilow and many others

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software for research (and education) in:
- geometric combinatorics: convex polytopes
- **linear/combinatorial optimization**
- algebraic geometry
- ...

open source, GNU Public License
- supported platforms: Linux, FreeBSD, MacOS X
- more than 100,000 uloc (Perl, C++, C, Java)

co-authored (since 1996) w/ Ewgenij Gawrilow [now TomTom]
- contributions by many people

www.polymake.org
Example: Maximal Matching

Let $G = (V, E)$ be a finite graph.

$$\max \sum_{e \in E} x_e$$

s.t. $\sum_{e \ni v} x_e \leq 1$ for all $v \in V$

$x_e \in \{0, 1\}$ for all $e \in E$
Algorithm Overview (Selection)

- convex polytopes, polyhedra and fans
  - convex hulls: cdd, lrs, beneath-and-beyond
  - Voronoi diagrams, Delone decompositions
  - face lattices: Kaibel–Pfetsch (including variations)
  - lattice polytopes/toric varieties

- optimization
  - Hilbert bases: normaliz, 4ti2
  - Gomory–Chvátal closures
  - counting integer points: LattE, bounding box/by projection

- graphs, matroids, ...
- simplicial complexes
- tropical geometry
Other polymake Resources for Optimization

- [http://polymake.org/tutorial/...](http://polymake.org/tutorial/...)
  - Marc Pfetsch & Sebastian Pokutta: optimization tutorial
  - Michael Schmitt: implementation of branch-and-bound as proof-of-concept
- Matthias Walter: total unimodularity test [Wed 10:30, H 3005]
Technical Aspects

- Hybrid design: Perl (interpreted) and C++ (compiled)
  - Perl: Server side (= organization/communication)
  - C++: Client side (= computation)
- Shell type user interface
  - (extension of) Perl as language
- Technical features include:
  - C++ template library
    - extends STL, based on template meta-programming
  - shared memory communication between client/server, transaction safe
  - whole system can be used as a C++ library (since 2.12)
- prototype: pypolymake [Burcin Erocal]
- interfaces to polymake in the making:
  - Singular, GAP, Sage
Objects and Properties

- hierarchy of **big object types** (modelling mathematical concepts)
  - e.g., polytopes, simplicial complexes, graphs, ...
  - under control of client/server system
  - with templates
- **properties** as class members (functions or data)
  - strongly typed
  - a type is a built-in Perl type, a C++ class type, or a big object type
  - immutable
- new big object types and properties to a given big object type *can be added at will*
- big object types grouped into **applications** (≈ name spaces)
New Features of polymake 2.13

- quadratic field extensions and exact representations of Platonic solids (and others)
- regularity for complete fans
- commutative algebra: new application ideal
- visualization: improved Sketch output
- fast loading of gzipped XML files
- further modularization via bundled extensions