polymake tutorial:
Which Convex Hull Algorithm Should I Use?

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joint w/ polymake team
polymake Overview

- software for research in mathematics
  - geometric combinatorics: convex polytopes, matroids, ...
  - linear/combinatorial optimization
  - toric/tropical geometry
  - combinatorial topology

- open source, GNU Public License
  - supported platforms: Linux, FreeBSD, MacOS X
  - about 100,000 uloc (C++, Perl, C, Java)

- co-authored (since 1996) w/ Ewgenij Gawrilow
  - contributions by Benjamin Assarf, Katrin Herr, Silke Horn, Lars Kastner, Georg Loho, Benjamin Lorenz, Andreas Paffenholz, Julian Pfeifle, Thomas Rehn, Thilo Rödig, Benjamin Schröter, and others
DEMO: (integer) linear programs
Facets of Cut Polytopes

variable dimension

\[ d = k + 6 \]
\[ n = 2^{k+5} = \# \text{ cuts} \]
\[ m = 2d + 8 = 2k + 20 \]

Barahona 1983: facets known if no \( K_5 \)-minor
DEMO: cut polytopes
Knapsack Polytop Integer Hulls

fixed dimension

$$a_1 = 2, \ a_2 = 3, \ a_i = a_{i-2} + a_{i-1}$$

$$F_d(b) = \{ x \in \mathbb{R}_{\geq 0}^d \mid a^T x \leq b \}$$

- $$d = 5$$
- $$n = 1366, 3173, 6509, 12182, 21245, 35025, 55157$$
- $$m = 12, 15, 12, 12, 8, 13, 15$$
DEMO: Voronoi diagrams
Voronoi Diagrams of Random Points in a Box

[Graphs showing random points and their Voronoi diagrams]
Some Rules of Thumb

1. If you do not know anything about your input, try double description.
   - cdd, ppl, nmz
2. Do use double description for computing the facets of 0/1-polytopes.
   - cdd, ppl
3. On random input beneath-and-beyond often behaves very well.
   - bb
4. Use reverse search for partial information and non-degenerate input.
   - lrs
EPILOGUE