Jörgshop TU Berlin, 2017

# Convex and Discrete Geometry

June 02, 2017 - June 03, 2017

# held in honour of the 80th birthday of Prof. Dr. Dr. h. c. **Jörg M. Wills**

### Program

All talks will be in room MA041 (ground floor) of the Mathematics Building at TU Berlin (Strasse des 17 Juni 136).

### Friday, June 2nd

10:00 Welcor
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- 10:10-11:00 Rolf Schneider On polyhedral cones 11:10-12:00 Günter M. Ziegler On 4-Polytopes and 3-Spheres
  - lunch & coffee break
- 14:00-14:50 Vitali Milman "Irrational" Convexity
- 15:00 15:50 Apostolos Giannopoulos *Quantitative versions of Helly's theorem* coffee break
- 16:30–17:20 Chuanming Zong The Sausage Conjecture
  - 17:30 in memoriam Peter M. Gruber19:30 social dinner at Zollpackhof

### Saturday, June 3rd

- 09:30-10:20 Jürgen Bokowski Methods for Geometric Realization Problems
  10:30-11:20 Károly Böröczky The Wills functional and translation covariant valuations lunch & coffee break
   13:30-14:20 Peter McMullen Interconnexions
- 14:30–15:20 Peter Gritzmann Diagrams and Democracy: Electoral District Design via Constrained Clustering good bye coffee

The restaurant Zollpackhof (social dinner) is in Elisabeth-Abegg-Str. 1, 10557 Berlin.

via S-Bahn: "Hauptbahnhof"

Bus: "Bundeskanzleramt" (TXL, 123): "Platz der Republik" (100) Ferry: "Paul-Löbe-Ufer", "Moltkebrücke"

### The Wills functional and translation covariant valuations

## Károly Böröczky

Budapest

*Abstract.* The talk reviews some results on translation covariant valuations where some fundamental examples originate from the Wills functional.

## Methods for Geometric Realization Problems Tested e.g. for Hurwitz's Regular Map (3,7)

### Jürgen Bokowski

Darmstadt

Abstract. Jörg tried to find a geometric realization in 3-space for Hurwitz's regular map (3,7). Compare a topological realization of J. J. van Wijk beside. This map has 72 points, 252 edges, and 168 triangles. Its symmetry group has order 1008. Every point is incident with seven triangles and every edge is incident with precisely two triangles. Such a realization in 3-space should show the 168 triangles of this regular map of genus seven without self-intersections. We use this example to describe methods for finding such realizations or to prove the non-existence.



Geometric realization problems appear in several other mathematical areas. We discuss such problems and some solutions.

### Quantitative versions of Helly's theorem

### Apostolos Giannopoulos

Athens

*Abstract.* We discuss a number of new quantitative versions of Helly's theorem. We provide improved, polynomial in the dimension, estimates for the volume radius, the diameter and other parameters of the intersection of a family of convex bodies. The results are mainly due to Silouanos Brazitikos and complement previous work of Bárány-Katchalski-Pach and Naszódi. The method is based on spectral sparsification and on an appropriate version of the Brascamp-Lieb inequality associated with an approximate decomposition of the identity.

Diagrams and Democracy: Electoral District Design via Constrained Clustering

## Peter Gritzmann

### Munich

*Abstract.* We study the electoral district design problem where municipalities of a state have to be grouped into districts of nearly equal population while obeying certain politically motivated requirements. We develop a general framework for electoral district design that is based on the close connection of constrained geometric clustering and diagrams. The approach is computationally efficient and flexible enough to pursue various conflicting juridical demands for the shape of the districts. We demonstrate the practicability of our methodology for electoral districting in Germany.

(Joint work with A. Brieden and F. Klemm)

### Interconnexions

## Peter McMullen

### London

*Abstract.* It makes a lot of sense to discuss a good part of the basic theory of (convex) polytopes in the wider context of polyhedra (polyhedral sets). In turn, their properties can be developed in quite a different way from those of general convex sets, particularly because of their specialized nature. As just one instance, the (Minkowski) sum of two polyhedra is always a polyhedron, whereas the sum of two unbounded closed convex sets need not be closed. Thus closure is one property that does not cause problems in the narrower context.

The idea behind the talk is not only to show how the theory can be put together in an alternative way, but also to point out how this leads to an early discussion of relationships among various basic concepts in the theory. These include fibre operations, polarity, finite tilings and strong duality, conjugacy relations for polyhedral functions, several kinds of diagram techniques, and hyperplane arrangements.

### "Irrational" Convexity

### Vitali Milman

## Tel Aviv

*Abstract.* Do we have enough examples of Convex Bodies? Is diversity of our standard examples enough to understand Convexity?

In the talk we demonstrate many different constructions which are analogous to constructions of irrational numbers from rationals. We show, following II. Molchanov, that the solutions of "quadratic" equations like  $Z^o = Z + K$  always exists (where  $Z^o$  is the polar body of Z; Z and K are convex compact bodies containing 0 in the interior). Then we show how the geometric mean may be defined for any convex compact bodies K and T (containing 0 into their interior).

We also construct  $K^a$  for any centrally symmetric K and 0 < a < 1, and also  $\log K$  for K containing the euclidean ball D (and K = -K). Of course,  $(K^a)^b = K^{(a,b)}$  and  $\log K^a = a . \log K$ . Note, the power a cannot be above 1 in the definition of power! It looks to be fundamental the notion of the Core of the body K is introduced.

(These results are joint with Liran Rotem.)

#### On polyhedral cones

## **Rolf Schneider**

## Freiburg

Abstract. Several recent applications of random models in convex optimization and signal demixing have brought into focus the following geometric question. "When does a randomly oriented cone strike a fixed cone?" More precisely, let C, D be polyhedral convex cones in  $\mathbb{R}^d$  and let  $\boldsymbol{\theta}$  be a uniform random rotation of  $\mathbb{R}^d$ ; what is the probability that  $C \cap \boldsymbol{\theta} D \neq \{o\}$ ? An explicit answer can be given if one knows the conic intrinsic volumes, the spherical kinematic formula, and the spherical Gauss–Bonnet theorem (here 'spherical' and 'conic' amounts essentially to the same). We review these topics and expand them by improvements and variants. Particular attention is payed to combinatorial aspects, valuations, and to polyhedral cones induced by central hyperplane arrangements.

### **On 4-Polytopes and 3-Spheres**

## Günter M. Ziegler

Berlin

Abstract. There has been massive efforts to understand the parameter spaces of convex polytopes – and great results such as the "g-Theorem" were achieved on the way. On the other hand, key questions are still open, already and in particular for the case of 4-dimensional polytopes/3-dimensional spheres. One crucial question is "fatness problem" for 4-dimensional polytopes, which is a key to the question whether we should expect the same answers for cellular spheres (a topological model) as for for convex polytopes (which are discrete-geometric objects), or for inscribable convex polytopes (which still have an extra condition). I will argue that we should not, and present first results in this direction:

The sets of *f*-vectors of 3-spheres and of 4-polytopes do not coincide.

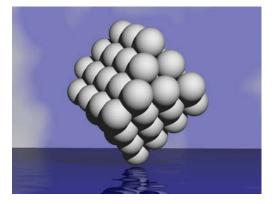
(Joint work with Philip Brinkmann and others)

### The Sausage Conjecture - An Excursion in Finite Packings

# Chuanming Zong

Beijing / Tianjin

Abstract. In 1975, L. Fejes Tóth studied the packing problem by m unit balls in n-dimensional space and proposed the sausage conjecture: Let  $\omega_n$  denote the volume of the n-dimensional unit ball. In n-dimensional Euclidean space with  $n \ge 5$ , the volume of the convex hull of m nonoverlapping unit balls is at least  $2(m-1)\omega_{n-1} + \omega_n$ , with equality being attained only when the centers of these balls are equally spaced a distance 2 apart on a line. In 1994, this conjecture was first proved for  $n \ge 13,387$  by U. Betke, M. Henk and J. M. Wills. In this talk, we will take an excursion in finite packings surrounding this conjecture.



ENJOY!