0/1-Polytopes: Random Walks and Phase-Transitions

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In the first part of the lecture, we will deal with a fascinating conjecture due to Mihalisin and Vazirani, stating that the graphs defined by the one-dimensional faces of 0/1-polytopes have excellent edge-expansion properties. We will explain the significance of this conjecture within the theory of randomized approximate counting and survey results supporting the conjecture, some of which recently have been obtained in joint work with Rafael Mechtel.

In the second part, we will present results on the numbers of low-dimensional faces of random 0/1-polytopes. The main result describes, for each constant k, a threshold function for the number of vertices, at which the expected k-face-density of a random d-dimensional 0/1-polytope shows a phase-transition from 'almost one' to 'almost zero'. Here, the k-face density of a polytope is the fraction of (k+1)-subsets of its vertices forming k-dimensional simplex-faces. These results in particular indicate that the high graph-densities often encountered for 0/1-polytopes associated with combinatorial optimization problems (e.g., cut-polytopes of complete graphs) seem to be due to the mere geometry of 0/1-polytopes rather than due to the underlying combinatorial structures.