The 0/1-Borsuk problem in low dimensions

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The 0/1-Borsuk problem asks whether any set of 0/1-vectors in \mathbb{R}^d can be partitioned into at most d + 1 sets of smaller diameter. This is known to be false in high dimensions (in particular for $d \geq 561$, due to Kahn & Kalai, Nilli, and Raigorodskii), which yields counterexamples to Borsuk's problem from 1933.

Here we ask whether there might be counterexamples in low dimension as well, and we show that there is no counterexample to the 0/1-Borsuk conjecture in dimensions $d \leq 9$. (In contrast, the general Borsuk conjecture is open even for d = 4.)