Lower bound for the maximal number of facets of a 0/1 polytope

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(joint work with D. Gatzouras and N. Markoulakis)

We will discuss a lower bound for the maximal possible number of facets of a 0/1 polytope in $\mathbb{R}^n$. Let

$$g(n) := \max\left\{f_{n-1}(P_n) : P_n \text{ is a } 0/1 \text{ polytope in } \mathbb{R}^n\right\}$$

where $f_{n-1}(\cdot)$ denotes the number of facets. Fukuda and Ziegler asked what the behaviour of $g(n)$ is as $n \to \infty$. The best known upper bound to date is $g(n) \leq 30(n-2)!$ (for n large enough), proved by Fleiner, Kaibel and Rote. In the other direction, a major breakthrough was made by Bárány and Pór in 2001; they proved that

$$g(n) \geq \left(\frac{cn}{\log n}\right)^{n/4}$$

where $c > 0$ is an absolute constant. We will show that the exponent $n/4$ can in fact be improved to $n/2$:

Theorem: *There exists a constant $c > 0$ such that*

$$g(n) \geq \left(\frac{cn}{\log^2 n}\right)^{n/2}.$$

The existence of 0/1 polytopes with many facets is established by a refinement of the probabilistic method developed by Bárány and Pór.