Supraconvergence of a finite difference scheme for elliptic third kind boundary value problems in fractional order Sobolev spaces

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Abstract

In this paper we study the convergence of a finite difference discretization of a second order elliptic equation with variable coefficients subject to general boundary conditions. We prove that the scheme exhibits the phenomenon of supraconvergence on non-uniform grids, i.e. although the truncation error is in general of first order only, one has second order convergence. More precisely, for $s \in (1/2, 2]$ optimal order $O(h^s)$-convergence of the finite difference solution and its gradient is shown if the exact solution is in the Sobolev-Slobodetskij space $H^{1+s}(\Omega)$. All error estimates are strictly local.

Another result of the paper is a close relationship of the finite difference scheme and a linear finite element method combined with a special kind of quadrature. As a consequence, the results of the paper can be viewed as introducing a fully discrete finite element method for which the gradient is superclose, i.e., the error of the approximate gradient with respect to a linear interpolation of the solution u is of second order if $u \in H^3(\Omega)$. A numerical example is given.

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