Supraconvergence and supercloseness of a scheme for elliptic equations on non-uniform grids

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Abstract

In this paper we study the convergence of a finite difference scheme on non-uniform grids for the solution of second order elliptic equations with mixed derivatives and variable coefficients in polygonal domains subject to Dirichlet boundary conditions. We show that the scheme is equivalent to a fully discrete linear finite element approximation with quadrature. It exhibits the phenomenon of supraconvergence, more precisely, for $s \in [1,2]$ order $O(h^s)$-convergence of the finite difference solution and its gradient is shown if the exact solution is in the Sobolev space $H^{1+s}(\Omega)$. In the case of an equation with mixed derivatives in a domain containing oblique boundary sections the convergence order is reduced to $O(h^{3/2-\epsilon})$ with $\epsilon > 0$ if $u \in H^3(\Omega)$. The second order accuracy of the finite difference gradient is in the finite element context nothing else than the supercloseness of the gradient. For $s \in \{1,2\}$ the given error estimates are strictly local.

Key words: finite element method, finite difference scheme, nonuniform grids, stability, supraconvergence, superconvergence of gradient.

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