

Supraconvergence of a finite difference scheme for solutions in $H^s(0, L)$

S. Barbeiro*, J.A. Ferreira † and R.D. Grigorieff ‡

Abstract

In this paper we study the convergence of a fully discrete linear finite element solution for a one-dimensional elliptic problem subject to general boundary conditions. We prove for $s \in [1, 2]$ order $O(h^s)$ convergence of solution and gradient if the exact solution is in the Sobolev space $H^{s+1}(0, L)$. The method is equivalent to a finite difference scheme on a nonuniform mesh and the obtained convergence is then a so-called supraconvergence result for solution and gradient. Numerical results illustrate the performance of the method and support the convergence result.

*Universidade de Coimbra, Faculdade de Ciências e Tecnologia, Departamento de Matemática, Apartado 3008, 3000 Coimbra, Portugal. email: silvia@mat.uc.pt

†Universidade de Coimbra, Faculdade de Ciências e Tecnologia, Departamento de Matemática, Apartado 3008, 3000 Coimbra, Portugal. email: ferreira@mat.uc.pt

‡Technische Universität Berlin, Straße des 17. Juni 135, 10623 Berlin, Germany. email-grigo@math.tu-berlin.de