On Qualocation and Collocation Methods for Singular Integral Equations with Piecewise Continuous Coefficients, Using Continuous Splines on Quasi-uniform Meshes

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Dedicated to the memory of Siegfried Prößdorf

Abstract

In this paper the qualocation method (which includes the collocation method as a special case) is applied to index-zero singular integral equations with piecewise-continuous coefficients, using continuous splines defined on a quasi-uniform mesh. Because the mesh is not diffeomorphic to a uniform mesh, Fourier series techniques are not available. Instead use is made of recent super-approximation results of Grigorieff, Sloan and Brandts for continuous splines on general meshes. The main result of the paper is that if a particular qualocation method is stable when applied to the identity operator, then the qualocation method is L_2 stable when applied to a singular integral equation if and only if the same method is L_2 stable when applied to all frozen-coefficient versions of the equation. The main theoretical tool is a local principle for splines in the form given by Prößdorf.

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