On Qualocation and Collocation Methods for Singular Integral Equations with Piecewise Continuous Coefficients, Using Continuous Splines on Quasi-uniform Meshes

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Dedicated to the memory of Siegfried Prößdorf

Abstract

In this paper the qualocation method (which includes the collocation method as a special case) is applied to index-zero singular integral equations with piecewise-continuous coefficients, using continuous splines defined on a quasi-uniform mesh. Because the mesh is not diffeomorphic to a uniform mesh, Fourier series techniques are not available. Instead use is made of recent superapproximation results of Grigorieff, Sloan and Brandts for continuous splines on general meshes. The main result of the paper is that if a particular qualocation method is stable when applied to the identity operator, then the qualocation method is $L_2$ stable when applied to a singular integral equation if and only if the same method is $L_2$ stable when applied to all frozen-coefficient versions of the equation. The main theoretical tool is a local principle for splines in the form given by Prößdorf.

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