

# Superapproximation and commutator properties of discrete orthogonal projections for continuous splines

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## Abstract

This paper builds upon the  $L_p$ -stability results for discrete orthogonal projections on the spaces  $S_h$  of continuous splines of order  $r$  obtained by Grigorieff and Sloan in [1]. Properties of such projections were proved with a minimum of assumptions on the mesh and on the quadrature rule defining the discrete inner product. The present results, which include superapproximation and commutator properties, are similar to those derived by Sloan and Wendland [2] for smoothest splines on uniform meshes. They are expected to have applications (as in [3]) to quallocation methods for non-constant-coefficient boundary integral equations, as well as to the wide range of other numerical methods in which quadrature is used to evaluate  $L_2$ -inner products. As a first application, we consider the most basic variable-coefficient boundary integral equation, in which the constant-coefficient operator is the identity. The results are also extended to the case of periodic boundary conditions, in order to allow application to boundary integral equations on closed curves.

- [1] R.D. Grigorieff and I.H. Sloan, *Stability of discrete orthogonal projections for continuous splines*, Bull. Austral. Math. Soc. 58, pp. 307–332, 1998.
- [2] I.H. Sloan and W. Wendland, *Commutator properties for periodic splines*, J. Approx. Theory, 97, pp. 254–281, 1999.
- [3] I.H. Sloan and W. Wendland, *Spline quallocation methods for variable-coefficient elliptic equations on curves*, Institut Mittag-Leffler, Report No. 5, 1997/1998, ISSN 1103-467X, Numer. Math., to appear.

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