Spline Petrov-Galerkin Methods with Quadrature

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Abstract

This paper studies certain discrete schemes for mth-order linear 2-point boundary value problems and related eigenvalue problems. The method is a discrete Petrov-Galerkin method, with a trial space of smoothest splines of order $m+2$, and a test space of continuous piecewise-linear functions. The method, which extends and generalises recent results of Sloan, Tran and Fairweather for the case $m=2$, places no restrictions on the mesh. Under appropriate smoothness assumptions, the order of convergence of the mth derivatives of the boundary-value problem is $O(h_{\max}^2)$, while that of the $(m-2)$th and lower derivatives is $O(h_{\max}^4)$. Order $O(h_{\max}^4)$ convergence is also proved for the eigenvalues in the case of the eigenvalue problem.

Remark: In the published version of the paper it is claimed that the $(m-1)$th derivative converges with order $O(h_{\max}^4)$ at the breakpoints, but this result does not hold true.

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