High-Order Spline Petrov-Galerkin Methods with Quadrature

R.D. Grigorieff[1] and I.H. Sloan[2]

Abstract

This note is concerned with a high-order spline Petrov-Galerkin method for m-th order two-point boundary value problems and generalizations in which the L^2-inner product is replaced by a composite quadrature rule. The elementary rule from which the composite rule is formed need not be accurate, but its weights are required to be positive, and the number of points cannot be smaller than a certain minimum value. Certain collocation methods are included as a special case. There are no restrictions on mesh ratios.

The stability and hence convergence of the methods in L^p-spaces depends on the uniform boundedness of a discrete analogue of the L^2 orthogonal projection on certain spline spaces with respect to the L^p-norm. Under additional assumptions on the quadrature rules, superconvergence results for the approximate solution and eigenvalues can be proved.

[1]Technische Universität Berlin, Strasse des 17. Juni 135, D-10623 Berlin, Germany. email: grigo@math.tu-berlin.de

[2]School of Mathematics, University of New South Wales, Sydney 2052, Australia. email: I.Sloan@unsw.edu.au