# Semi-Discrete Galerkin Approximation of the Single Layer Equation by General Splines

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**Abstract**

This paper deals with a semi-discrete version of the Galerkin method for the single-layer equation in a plane, in which the outer integral is approximated by a quadrature rule. A feature of the analysis is that it does not require high precision quadrature or exceptional smoothness of the data. Instead, the assumptions on the quadrature rule are that constant functions are integrated exactly, that the rule is based on sufficiently many quadrature points to resolve the approximation space, and that the *Peano constant* of the rule is sufficiently small. It is then shown that the semi-discrete Galerkin approximation is well posed. For the trial and test spaces we consider quite general piecewise polynomials on quasi-uniform meshes, ranging from discontinuous piecewise polynomials to smoothest splines. Since it is not assumed that the quadrature rule integrates products of basis functions exactly, one might expect degradation in the rate of convergence. To the contrary, it is shown that the semi-discrete Galerkin approximation will converge at the same rate as the corresponding Galerkin approximation in the $H^0$ and $H^{-1}$ norms.

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