

# Stability of Discrete Orthogonal Projections for Continuous Splines in $L_p$ -Spaces

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## Abstract

In this paper  $L_p$  stability and convergence properties of discrete orthogonal projections on the finite element space  $S_h$  of continuous polynomial splines of order  $r$  are proved. The discrete inner products are defined by composite quadrature rules with positive weights on a sequence of nonuniform grids. It is assumed that the basic quadrature rule  $Q$  has at least  $r$  quadrature points in order to resolve  $S_h$ , but no accuracy is required. The main results are derived under minimal further assumptions, for example the rule  $Q$  is allowed to be non-symmetric, and no quasi-uniformity of the mesh is required. The corresponding stability of the orthogonal  $L_2$ -projections has been studied by de Boor [1] and by Crouzeix and Thomée [2]. Stability of the first derivative of the projection is also proved, under an assumption (unless  $p = 1$ ) of local quasi-uniformity of the mesh.

- [1] C. de Boor: A bound on the  $L^\infty$ -norm of  $L^2$ -approximation by splines in term of a global mesh ratio. *Math. Comp.* **30**, 765-771 (1976).
- [2] M. Crouzeix and V. Thomée: The stability in  $L_p$  and  $W_p^1$  of the  $L_2$ -projection onto finite element function spaces. *Math. Comp.* **48**, 521-532 (1987).

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