# Tolerance Graphs and Orders*

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**Abstract.** We show that if a tolerance graph is the complement of a comparability graph it is a trapezoid graph, i.e., the complement of an order of interval dimension at most 2. As consequences we are able to give obstructions for the class of bounded tolerance graphs and to give an example of a graph which is alternatingly orientable but not a tolerance graph. We also characterize the tolerance graphs among complements of trees

# 1 Introduction and Overview

An undirected graph $G = (V, E)$ is called a *tolerance graph* if there exists a collection $\mathcal{I} = \{I_x \mid x \in V\}$ of closed intervals on the line and a (tolerance) function $t : V \to \mathbb{R}^+$ satisfying

$$\{x, y\} \in E \iff |I_x \cap I_y| \geq min(t_x, t_y)$$

where $|I|$ denotes the length of the interval $I$. A tolerance graph is a *bounded tolerance graph* if it admits a tolerance representation $\{\mathcal{I}, t\}$ with $|I_x| \geq t_x$ for all $x \in V$.

Tolerance graphs were introduced by Golumbic and Monma [7]. They show that if all tolerances $t_x$ equal the same value $c$, say, then we obtain exactly the class of all interval graphs. If the tolerances are $t_x = |I_x|$ for all vertices $x$, then we obtain exactly the class of all permutation graphs. Furthermore, the following theorem was proved.

**Theorem 1** *Every bounded tolerance graph is the complement of a comparability graph, i.e., a cocomparability graph.*

The most important article on tolerance graphs is due to Golumbic, Monma and Trotter [8]. We summarize some of the results shown there in the next theorem.

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**Theorem 2**

*(1) A tolerance graph must not contain a chordless cycle of length greater than or equal to 5.*
*(2) A tolerance graph must not contain the complement of a chordless cycle of length greater than or equal to 5.*
*(3) A tolerance graph admits an orientation such that every chordless 4 cycle is oriented as shown in Figure 1.*

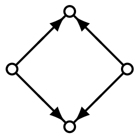

Figure 1: Alternating orientation of $C_4$

A graph $G$ is called *alternatingly orientable* if there is an orientation of $G$ such that around every chordless cycle of length greater than 3 the directions of arcs alternate. As a consequence of the preceding theorem, we obtain that tolerance graphs are alternatingly orientable, (see [3] for more information on this class of graphs). We report on some results related to the following open problems.

**Problem 1** *Characterize tolerance and bounded tolerance graphs.*

**Problem 2** *Is the intersection of tolerance graphs and cocomparability graphs exactly the class of bounded tolerance graphs?*

# 2 Tolerance Graphs and Orders of Interval Dimension 2

The starting point of this work is a representation theorem for bounded tolerance graphs. Let $G = (V, E)$ be a bounded tolerance graph with representation $\{\mathcal{I}, t\}$ and $I_x = [a_x, b_x]$. We define two interval orders $P^1, P^2$ on the set of vertices of $G$. Let $P^1$ be represented by the intervals $I_x^1 = [a_x + t_x, b_x]$ and let $P^2$ be represented by the intervals $I_x^2 = [a_x, b_x - t_x]$. We claim that $G$ is the cocomparability graph of $P = P^1 \cap P^2$.

Let vertices $x$ and $y$ be joined by an edge in $G$. Assume $x < y$ in $P^1 \cap P^2$. It follows that $b_x < a_y + t_y < b_y$ and $a_x < b_x - t_x < a_y$. In the given relative position of intervals $|I_x \cap I_y| = b_x - a_y$. From the previous inequalities $b_x - a_y < t_x$ and $b_x - a_y < t_y$. This contradicts the edge condition $|I_x \cap I_y| \geq min(t_x, t_y)$.

Since $P$ is the intersection of two interval orders $P$ we have shown that $G$ is the complement of the comparability graph of an order with interval dimension at most 2.

A special feature of the interval realizer $\{\mathcal{I}^1, \mathcal{I}^2\}$ of $P$ is that $|I_x^1| = |I_x^2|$ for all $x \in V$. In the spirit of the term box embedding introduced in [6], we call such a representation a *square embedding.*

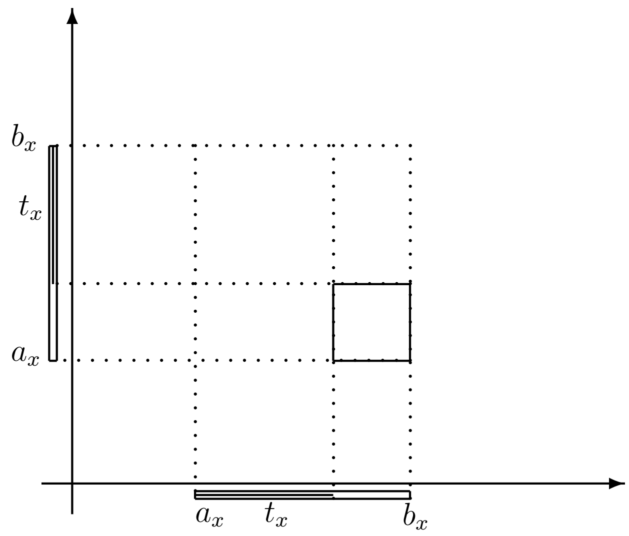


Figure 2: The square corresponding to $[a_x, b_x]$ and tolerance $t_x$

Let $P = (X, <)$ be an order of interval dimension $\leq 2$ that admits a square embedding. We claim that the cocomparability graph of $P$ (denoted by $CoComp(P)$) is a bounded tolerance graph. Let a square embedding of $P$ be given by $\{\mathcal{I}^1, \mathcal{I}^2\}$ and let the corresponding intervals of $x$ be given by $I_x^1 = [a_x^1, a_x^1 + l_x]$ and $I_x^2 = [a_x^2, a_x^2 + l_x]$. We now fix some $s \in I\!\!R$ such that $s \geq max_{x \in X}(\ a_x^2 - a_x^1\ )$. The cocomparability graph of $P$ is the bounded tolerance graph given by the intervals $I_x = [a_x^2, s + a_x^1 + l_x]$ and the tolerances $t_x = s + a_x^1 - a_x^2$. This proves the following characterization of bounded tolerance graphs.

**Theorem 3** *A graph $G$ is a bounded tolerance graph iff $G$ is the cocomparability graph of an order $P$ with interval dimension at most 2 which has a square embedding.*

Cocomparability graphs of orders with interval dimension at most 2 are known as *trapezoid graphs.* An observation similar to ours is used by Bogart et al. [2] to show that the class of bounded tolerance graphs coincides with the class of parallelogram graphs, i.e., trapezoid graphs where every trapezoid is a parallelogram.

There exist orders of interval dimension 2 which do not admit a square embedding. This is shown with the following example.

**Example.** It is easy to see that the graph $G$ given in Figure 3 is not alternatingly orientable. An orientation which is alternating on the cycles $(3,4,7,8)$, $(7,8,5,6)$, $(5,6,1,2)$ and $(1,2,4,3)$ and contains $3 \to 4$ would require $7 \to 8, 5 \to 6, 1 \to 2, 4 \to 3$ a contradiction. Therefore $G$ is not a tolerance graph.

On the other hand, $G$ is the cocomparability graph of the order $P$ whose box embedding is shown in Figure 3. The existence of a box embedding for $P$ proves that $Idim(P) = 2$.

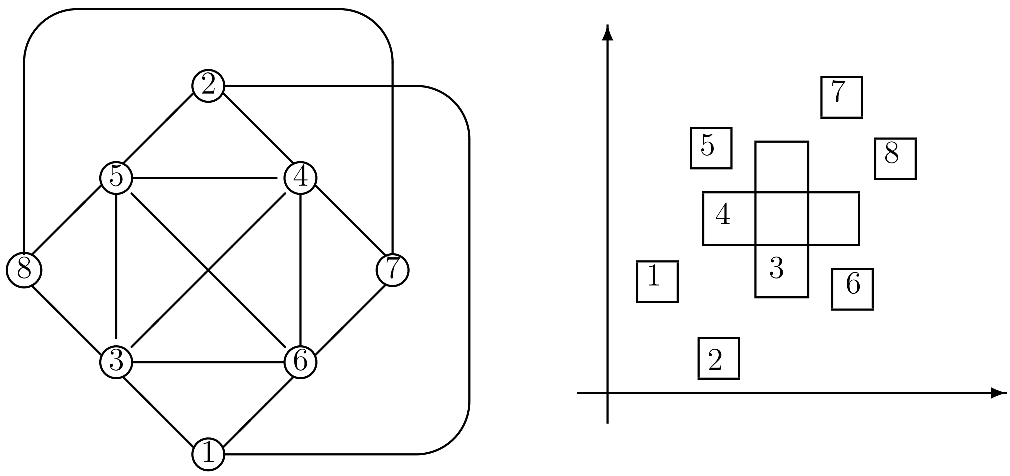


Figure 3: The graph $G = CoComp(P)$ and the order $P$.

As a consequence of the next theorem, we will obtain: *Every tolerance graph which is a cocomparability graph is a trapezoid graph.* This will be useful later when we characterize tolerance graphs among complements of trees and seems interesting also in view of Problem 2.

**Theorem 4** *The intersection of cocomparability graphs and alternatingly orientable graphs is contained in the class of trapezoid graphs.*

**Proof.** The main ingredient into the proof of this theorem will be the necessary condition for interval dimension 2 given in Lemma 1 below.

Let $P = (X, <)$ be an order of interval dimension two and $x < y$, $z < t$ be a **2+2** in $P$. The pairs $(x, t)$ and $(z, y)$ are the *diagonals* of the **2+2**. Let $\{\mathcal{I}^1, \mathcal{I}^2\}$ be an interval realizer of $P$. W.l.o.g. we may assume that $x \not< t$ in $\mathcal{I}^1$ and note that this implies $z < y$ in $\mathcal{I}^1$. In $\mathcal{I}^2$ we then have $z \not< y$ and $x < t$.

Associate with $P$ the *incompatibility graph* $F_P$.

- As vertices of $F_P$ we take the ordered incomparable pairs, i.e., $(x, y)$ and $(y, x)$ for all $x||y$.

- Two vertices of $F_P$ are connected by an edge iff they are the diagonals of a common **2+2** in $P$.

**Lemma 1** *If Idim$(P) \leq 2$ then $F_P$ is bipartite.*

*Proof.* We have shown that independent of the interval realizer $\{\mathcal{I}^1, \mathcal{I}^2\}$ a diagonal $(x,t)$ of a **2+2** is a relation in exactly one of $\mathcal{I}^1$ and $\mathcal{I}^2$. Color diagonal $(x,t)$ with color 1 if it is a relation in $\mathcal{I}^1$ and color all the remaining vertices of $F_P$ with color 2. This is a legal 2 coloring of $F_P$. △

We are ready to show that *Idim*$(P) > 2$ implies that the cocomparability graph of $P$ is not alternatingly orientable.

Let $P$ with *Idim*$(P) \geq 3$ be given. From Lemma 1 we know that $F_P$ contains odd cycles. Fix an odd cycle $C = [(x_1,y_1),(x_2,y_2),\ldots,(x_{2k+1},y_{2k+1}),(x_1,y_1)]$ in $F_P$. If $(x_i,y_i)$ and $(x_{i+1},y_{i+1})$ are consecutive elements of $C$ then, by the definition of $F_P$, $[x_i,y_i,y_{i+1},x_{i+1},x_i]$ is a 4-cycle in *CoComp*$(P)$. Therefore:

(∗) An alternating orientation of *CoComp*$(P)$ will either contain the two arcs $x_i \to y_i$ and $y_{i+1} \to x_{i+1}$ or the two arcs $y_i \to x_i$ and $x_{i+1} \to y_{i+1}$.

Suppose that an alternating orientation $A$ of *CoComp*$(P)$ is given. We may assume that $x_1 \to y_1$ is in $A$. Using (∗) we obtain that $y_2 \to x_2$ is in $A$. Using (∗) once more we obtain that $x_3 \to y_3$ is in $A$. Repeating this argument we finally find $x_{2k+1} \to y_{2k+1}$ and hence $y_1 \to x_1$ in $A$. This contradicts the existence of an alternating orientation. □

**Remark.** Cogis [4] has shown that the implication of Lemma 1 is in fact an equivalence. He obtained this result in the more general context of Ferrers-dimension for directed graphs.

## 2.1 Some Examples

In this section we give examples separating several classes of graphs. We first show that the class of graphs obtained as intersection of cocomparability graphs with alternatingly orientable graphs and the intersection class of cocomparability graphs with tolerance graphs are not the same.

Let $P = (X,<)$ be an order of interval dimension 2 with realizer, i.e., box embedding, $I_1 = \{\ [a^1_x, b^1_x]\ :\ x \in X\}$, $I_2 = \{\ [a^2_x, b^2_x]\ :\ x \in X\}$. We say $x, y \in X$ have *crossing diagonals* if the line segments $(a^1_x, a^2_x) \to (b^1_x, b^2_x)$ and $(a^1_y, a^2_y) \to (b^1_y, b^2_y)$ intersect in $\mathbb{R}^2$.

**Lemma 2** *If an order $P$ of interval dimension 2 has a box embedding without crossing diagonals then $G =$ CoComp$(P)$ has an alternating orientation.*

**Proof.** The orientation we define for $G$ will capture a relation of 'being left of' for pairs of diagonals. The non-crossing condition will be sufficient to have this relation well defined. As a cocomparability graph $G$ is free of cycles of length 5 and more we only have to show that the four-cycles in $G$ are alternatingly oriented. A four-cycle $(v, w, x, y)$ in $G$ corresponds to a **2+2** in $P$. Assume $v < x$ and $w < y$ and note that by an argument from the proof of Theorem 4 in every box embedding of $P$ the boxes of one edge of the **2+2** are left of the boxes of the other edge. If $v, x$ is left of $w, y$ we obtain the orientation $v \to w \leftarrow x \to y \leftarrow v$, i.e., an alternating orientation.

We now give a formal definition of the orientation on $G$. Let the Euclidean distance in the plane be $d[(u_1, v_1), (u_2, v_2)] = \sqrt{(u_1 - u_2)^2 + (v_1 - v_2)^2}$. Relative to the box of a vertex $x$ we define two regions $R^1(x)$ and $R^2(x)$ (see Figure 4).

$$R^1(x) = \{\ (u, v)\ :\ u \geq a_x^1 \text{ and } v \leq b_x^2 \text{ and } d[(u, v), (b_x^1, a_x^2)] \leq d[(u, v), (a_x^1, b_x^2)]\ \}$$
$$R^2(x) = \{\ (u, v)\ :\ u \leq b_x^1 \text{ and } v \geq a_x^2 \text{ and } d[(u, v), (b_x^1, a_x^2)] \geq d[(u, v), (a_x^1, b_x^2)]\ \}.$$

Note that if $y$ is incomparable to $x$ then the diagonal of the box of $y$ has to intersect

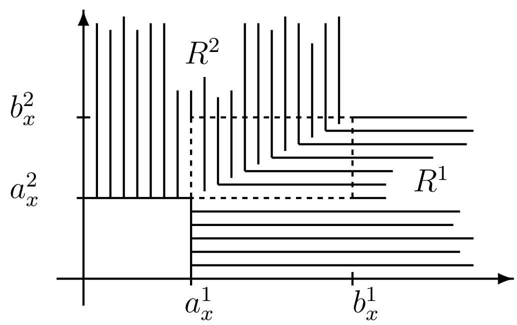


Figure 4: The regions defined by a box.

either $R^1(x)$ or $R^2(x)$. It can not intersect both since there are no crossing diagonals. If the diagonal of $y$ intersects $R^1(x)$ we orient the edge $\{x, y\}$ from $x$ to $y$ and if the diagonal of $y$ intersects $R^2(x)$ we orient it from $y$ to $x$. Obviously, taking the box of $y$ as the reference box for the definition of the orientation of edge $\{x, y\}$ leads to the same orientation. □

We will use Lemma 2 for the construction of an order $P$ such that $G = CoComp(P)$ is alternatingly orientable but $P$ admits no square embedding, i.e., $G$ is not bounded tolerance. The idea for the construction is to set up a box-order $P$, i.e, an order of interval dimension two, so that in every box realizer of $P$ four boxes $a, b, c, d$ form a 'windmill configuration' as in Figure 5. This implies that $P$ is not a square order. On the other hand Figure 5 shows a representation of $P$ without crossing diagonals.

The order $P$ consists of two parts. The first part to be defined will serve as a frame. The frame consists of chains $x_0 < x_1 < \ldots < x_4$ and $y_0 < y_1 < \ldots < y_4$ together with an element $z > x_0, y_0$. Let $I_1, I_2$ be an interval realizer of $F$. Since $x_0 < x_4$, $y_0 < y_4$ is a **2+2** in $F$ we have the relation $y_0 < x_4$ in either $I_1$ or $I_2$. Denote relations in $I_1$ by $<_1$ and relations in $I_2$ by $<_2$ and suppose $y_0 <_1 x_4$. This forces $x_0 <_2 y_4$. Relation $y_0 <_1 x_4$ also forces $x_3 <_2 y_1$ via the **2+2** $x_3 < x_4$ and $y_0 < y_1$. Choosing an appropriate sequence of induced **2+2** we find that $y_0 <_1 x_4$

forces $y_3 <_1 z$ and $z <_1 x_2$ and $y_3 <_1 x_1$. Symmetrically, $x_0 <_2 y_4$ forces $x_3 <_2 z$ and $z <_2 y_2$ and $x_3 <_2 y_1$. Hence, in every realizer chains $x_1 < x_2 < x_3$ and $y_1 < y_2 < y_3$ induce a 'grid-structure' on the quarterplane of points dominating the box of $z$.

Let $x^2$ be an element whose relations to the frame elements are $y_4 < x^2$, $x_1 < x^2$, $z < x^2$ together with the relations implied by transitivity. Let $x^3$, $y^2$ and $y^3$ be three more elements with relations $y_4 < x^3$, $x_2 < x^3$, $z < x^3$, $x_4 < y^2$, $y_1 < y^2$, $z < y^2$, $x_4 < y^3$, $y_2 < y^3$ and $z < y^3$. Note that the intervals of $x_2$ and $x^2$ and the intervals of $x_3$ and $x^3$ overlap in $I_1$. Similarly the intervals of $y_2$ and $y^2$ and the intervals of $y_3$ and $y^3$ overlap in $I_2$.

To obtain $P$ as shown in Figure 5 we add four more elements to form the windmill configuration. Let $a$, $b$, $c$ and $d$ be pairwise incomparable elements and relate them to the frame by $z < a, b, c, d$ and $x_0, y_1 < a$ and $a < x^3, y^2, y^3$ and $x_1, y_1 < b$ and $b < x^2, x^3$ and $x_1, y_2 < c$ and $c < y^3$ and $x_2, y_0 < d$ and $d < x^3, y^3$. Consider the intervals corresponding to $a$ in $I_1$ and note that it has to intersect the intervals of $x_1$ and $x^2$. Since $x_1 < b < x^2$ we find the interval of $b$ completely contained in the interval of $a$, i.e., the width of the box of $b$ is smaller than the width of the box of $a$. Similar arguments show that the height of the box of $c$ is smaller than the height of the box of $b$, the width of the box of $d$ is smaller than the width of the box of $c$ and finally, the height of the box of $a$ is smaller than the height of the box of $d$. Together this proves that there is no square embedding of $P$.

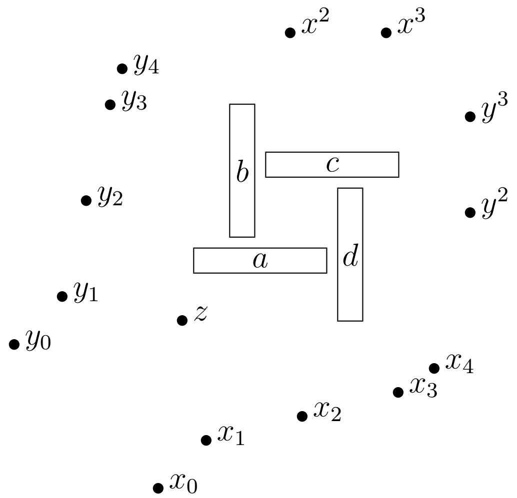


Figure 5: The complement of this order is alternatingly orientable but not a bounded tolerance graph

In Theorem 6 we will give more obstructions for the class of bounded tolerance graphs. Our next aim is to exhibit cocomparability graphs which possess an alternating orientation but are not tolerance graphs. To this end we introduce another notion.

A vertex $x$ of $G$ is called *assertive* if for every tolerance representation $\{\mathcal{I}, t\}$ of $G$ replacing $t_x$ by $min(t_x, |I_x|)$ leaves the tolerance graph unchanged. An assertive vertex never requires unbounded tolerance. Therefore, if every vertex of a tolerance graph $G$ is assertive then $G$ is bounded tolerance. In [8] it is shown that for a vertex $x$ to be nonassertive there has to be a $y$ with $\{x, y\} \notin E$ and $Adj(x) \subseteq Adj(y)$. Restating this we obtain.

**Lemma 3** *Let $x$ be a vertex in a tolerance graph $G = (X, E)$. If $Adj(x) \setminus Adj(y) \neq \emptyset$ for all $y$ with $\{x, y\} \notin E$, then $x$ is assertive.*

Given a graph $G = (X, E)$ we define a new graph $2G$. The set of vertices of $2G$ is the union of two copies $X_1, X_2$ of $X$. For all $x \in X$ there is an edge $\{x_1, x_2\}$ between the two copies of $x$ and for all $\{x, y\} \in E$ there are the four edges $\{x_i, y_j\}$ between copies of $x$ and $y$ in $2G$. That is, we substitute a 2 clique for each vertex of $G$ to obtain $2G$.

**Lemma 4** *If $G$ is not bounded tolerance then $2G$ is not a tolerance graph.*

**Proof.** Let $x, y$ be a pair of vertices of $2G$ with $\{x, y\} \notin E$ then $x' \in Adj(x) \setminus Adj(y)$ where $x'$ and $x$ are the two copies of an element of $G$. Therefore, if $2G$ is a tolerance graph it will only have assertive vertices and hence be a bounded tolerance graph. This, however, is impossible since $G$ is a subgraph of $2G$ and not even $G$ is bounded tolerance. □

Let $G$ be the cocomparability graph of the order of Figure 5 It is an easy exercise to show that the order corresponding to $2G$ has a box representation without crossing diagonals. Hence $2G$ is a cocomparability graph admitting an alternating orientation but not a tolerance graph.

## 2.2 Cotrees and More Examples

Complements of trees are cocomparability graphs. In [8] it is suggested to take them as an initial step towards a solution of Problem 2.

**Theorem 5** *If $\overline{T}$ is the complement of a tree $T$, then the following conditions are equivalent:*

*(1)* $\overline{T}$ *is a tolerance graph.*
*(2)* $\overline{T}$ *is a bounded tolerance graph.*
*(3)* $\overline{T}$ *is a trapezoid graph.*
*(4)* $T$ *is a tolerance graph.*
*(5)* $T$ *contains no subtree isomorphic to the tree $T_3$ of Figure 7.*

**Proof.** If $\overline{T}$ is a tolerance graph then, by Theorem 4, $T$ is the comparability graph of an order of interval dimension 2, i.e., $\overline{T}$ is a trapezoid graph. Orders which have trees as comparability graphs are of height 1. In a box realizer of a height 1 order

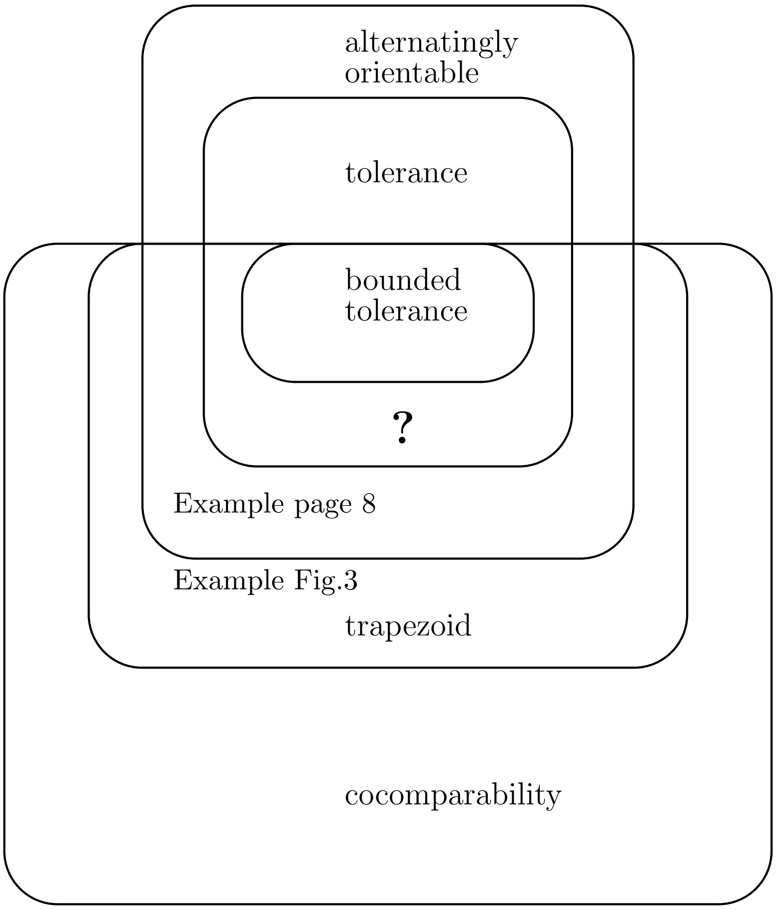


Figure 6: Relations of tolerance graphs to other classes of graphs.

with interval dimension 2 we have two types of move that do not change the order relations. First, we may move the upper right corner of boxes of maximal elements up and right and second, we may move the lower left corners of boxes of minimal elements down and left. Using this two moves we can transform all the boxes into squares. Therefore if $\overline{T}$ is a tolerance graph, then $\overline{T}$ is a bounded tolerance graph.

If $\overline{T}$ is a bounded tolerance graph, then trivially it is a tolerance graph, and by Theorem 3 it is a trapezoid graph. This gives the equivalence of (1),(2) and (3).

For the equivalence of (3) and (5) we need a characterization of those trees which are the comparability graphs of orders of interval dimension 2. In [6] it is shown that the interval dimension of a tree $T$ equals the dimension of the truncation of $T$, i.e., of the tree obtained after removing the leaves of $T$. Among the irreducible orders of dimension 3 there is only one tree, namely $T_2$. From this we obtain that $T_3$ is the unique tree among the obstructions against interval dimension 3. This last result can also be found in [10].

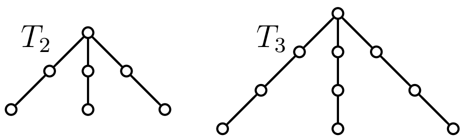


Figure 7: The trees $T_2$ and $T_3$.

For the remaining equivalence, i.e., to show the equivalence of (4) with (5) we refer to [8]. □

It was observed by Bogart and Trenk [1] that the above proof for the equivalence of (1) and (2) holds true in the more general case that $\overline{T}$ is the complement of a bipartite graph.

We now come back to minimal obstructions for the class of bounded tolerance graphs. Quite simple observation will provide us with many examples.

A now classical result of Dushnik and Miller says that a graph $G$ is both a comparability graph and a cocomparability graph exactly if $G$ and $\overline{G}$ are both comparability graphs of orders of dimension 2. An order $P$ is called *3–irreducible* if $dim\ P = 3$ but whatever vertex $x$ we remove from $P$ we obtain an order of dimension 2, i.e., $dim(P_x) = 2$.

Now let $P$ be 3–irreducible and $G = Comp(P)$. $G$ is not a bounded tolerance graph since it is not a cocomparability graph. But if we remove any vertex $x$ from $G$ then by the Dushnik and Miller characterization $G_x$ will be the cocomparability graph of an order of dimension 2. This order has an embedding by points, hence a square embedding by minisquares. This proves that $G_x$ is a bounded tolerance graph.

**Theorem 6** *If $P$ is a 3–irreducible order then the comparability graph of $P$ is a minimal obstruction for the class of bounded tolerance graphs.*

**Remark.** A complete list of 3–irreducible orders has independently been compiled by Kelly and by Trotter and Moore (see [9]). They found 10 isolated examples and 7 infinite families.

We now turn to a second large class of obstructions. Recall that in the proof of Theorem 5 we gave evidence that a height 1 order of interval dimension 2 admits a square embedding, i.e., its cocomparability graph is a bounded tolerance graph. An order $P$ is called *3–interval irreducible* if $Idim(P) = 3$ but whatever vertex $x$ we remove from $P$ we obtain an order of interval dimension 2, i.e., $Idim(P_x) = 2$.

Now let $P$ be a 3–interval irreducible order of height 1 and $G = CoComp(P)$. From $Idim(P) = 3$ it follows by Theorem 4 that $G$ is not a tolerance graph. But if we remove any vertex $x$ from $G$ then $G_x$ is the cocomparability graph of an order possessing a square embedding. Hence $G_x$ is a bounded tolerance graph.

**Theorem 7** *If $P$ is a 3–interval irreducible order of height 1 then the cocomparability graph of $P$ is a minimal obstruction for both the class of tolerance graphs and the class of bounded tolerance graphs.*

**Remark.** A complete list of the 3–interval irreducible orders of height 1 has been compiled by Trotter [10]. There are 3 isolated examples and 6 infinite families.

We close with a last example. Let $N(x) = Adj(x) \cup \{x\}$ denote the *neighborhood* of a vertex $x$ in $G$. A set of vertices $\{x_1, x_3, x_3\}$ is called an *asteroidal triple* if any two of them are connected by a path which avoids the neighbourhood of the remaining vertex. In [8] it is shown that cocomparability graphs do not contain asteroidal triples, hence bounded tolerance graphs are asteroidal triple-free as well. More information on asteroidal triple-free graphs is given in [5].

All examples of tolerance graphs which are not bounded tolerance graphs given in [8] contain an asteroidal triple. Therefore, it seems plausible to conjecture that every tolerance graph which is not bounded contains an asteroidal triple. Using Theorem 6 we now show that this is not true in general.

**Example.** Let $G$ be the comparability graph of the order $H_0$ from the list of 3–irreducible orders, see Figure 8. This graph is a tolerance graph and asteroidal triple-free, but by Theorem 6 it can not be a bounded tolerance graph.

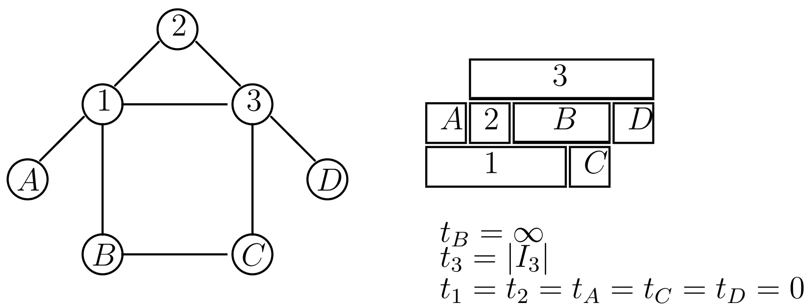


Figure 8: $G = Comp(H_0)$ and a tolerance representation of $G$.

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