10. Exercise sheet **Topics in Discrete Mathematics**
Stefan Felsner **Summer 2026**
Due for the exercise session: July 9, 2026

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(1) Let $\mathcal{A}$ be an arrangement of n pseudolines with a bounded cell. Prove that $\mathcal{A}$ has at least $2n/3$ triangles. Hint: If each pseudoline is incident to two triangles, then there are at least $2n/3$ triangles.

(2) How many triangles can an arrangement of n pseudolines maximally have? Determine the right order of magnitude.

(3) Let $\mathcal{A}$ be an arrangement of pseudolines, think of $\mathcal{A}$ as being represented by a wiring diagram. Let e be an edge of $\mathcal{A}$ supported by p_i and bounded by the crossings of p_i with p_j and p_k. If e is incident to a cell c, then we label e inside c with a $+$ if the triangle formed by p_i, p_j, p_k contains c. Show that

- A triangle contains three $+$ labels.
- A bounded cell of degree > 3 has at most two $+$ labels.
- An unbounded cell has no $+$ label.

(4) Use the previous exercise to derive a lower bound for the number of triangles in an arrangement of n pseudolines.

(5) Let $\mathcal{A}$ be a non-simple arrangement of lines with at least two crossing points. Show that $\mathcal{A}$ contains a crossing point where exactly two lines cross.

(6) The same as the previous exercise except that this time we look at an arrangement of pseudolines. Hint: Let T_j be the number of points where exactly j pseudolines cross and p_k be the number of faces of degree k. Consider the quantity

$$Y = \sum_{j \geq 2}(3-j)t_j + \sum_{j \geq 2}(3-k)p_k.$$