9. Exercise sheet **Topics in Discrete Mathematics**
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Due for the exercise session: July 2, 2026

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(1) Two zones of a rhombic tiling of a $2n$-gon can share only one rhombus. Provide a precise geometric proof.

(2) Show that the rhombi of a rhombic tiling of a $2n$-gon can be colored with three colors such that rhombi sharing an edge have different colors (see the picture on the hompace of the course).

(3) A circle containment graph is a geometric intersection graph where vertices represent circles (disks) in the plane, and an edge exists between two vertices if and only if one circle is strictly contained within the other. Show that the number of circle containment graphs on $V = [n]$ is asymtotically much smaller then the number of graphs on this vertex set.

(4) Let $\mathcal{H}$ be an arrangement of hyperplanes in $\mathbb{R}^d$. We assume that $\mathcal{H}$ is simple, i.e., each vertex of the arrangement is incident to exactly d hyperplanes. Show that the number Z of full dimensional cells is

$$Z = \binom{n}{d} + \binom{n}{d-1} + \binom{n}{d-2} + \cdots + \binom{n}{1} + \binom{n}{0}.$$

(5) Let $\mathcal{A}$ be an arrangement of n pseudolines with a bounded cell. Prove that $\mathcal{A}$ has at least $2n/3$ triangles. Hint: If each pseudoline is incident to two triangles, then there are at least $2n/3$ triangles.

(6) How many triangles can an arrangement of n pseudolines have? Determine the right order of magnitude.