6. Exercise sheet **Topics in Discrete Mathematics**
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Due for the exercise session: June 11, 2026

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(1) Can the 8×8 checkerboard be tiled with 15 Ts and a 2×2 square?

(2) A *break-line* in a tiling of a grid region is a horizontal or vertical line such that none of the tilles of the tiling crosses the line.

a. Show that every tiling of a 6×6 board with dominos has a break-line.

b. Find a tiling of the 8×8 board with Ts that has no break-line.

(3) A L-triomino is obtained from a 2×2 by removing one of its four squares.

a. Which $n \times n$ boards can be tiled by L-triominoes?

b. Which $n \times n$ boards with a removed corner square can be tiled by L-triominoes?

c. If we scale an L-triomino by a positive integer, it is a scaled L-triomino. Which scaled L-triominoes can be tiled by L-triominoes?

(4) Let Ω be the set of all domino tilings of the aztec diamond $A(n)$. A flip is the exchange of two horizontal dominos covering a 2×2 square by a pair of vertical dominos or vice versa. Show that it is possible to commute between any two tilings in Ω via flips.

(5) We look at tilings of a triangular grid with rhombii, i.e., unions of two triangles of the grid sharing an edge. The region is triangular grid with sidelength n, i.e., with $n + 1$ vertices on each side, and n holes. A hole is a triangle in the grid which has the same orientation as the full region, i.e., no two holes share an edge. Holes are not covered by rhombi.

a. Show that a tiling can only exist if each subtriangle of sidelength k contains at most k holes.

b. Show that the condition of **a.** is also sufficient for the existence of a tiling.