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Happy Endings for Flip Graphs

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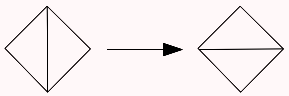
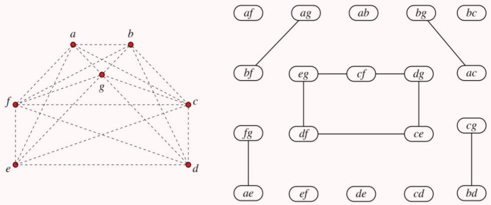
Introduction

Flip: 

Flip graph G

Flip distance: d_G

Quadrilateral graph $QG\ P$:



- Vertices: Dianonals of P
- Edge ab-cd: ab, cd cross and {a,b,c,d} form an empty quadrilateral

Notice: P is not necessarily in general position.

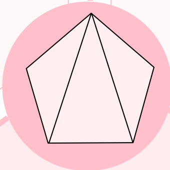
Pentagons and partial cubes

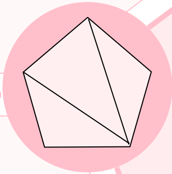
Let P be a finite point set. Equivalent:

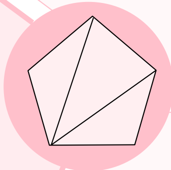
- P has no empty pentagon
- $QG\ P$ is a forest
- The flipgraph of P is a partial cube

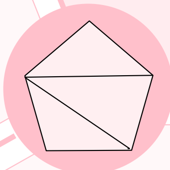
Finding pentagons and flips

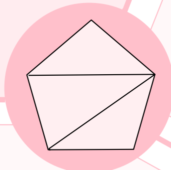
- One may test whether any set of n points contains an empty pentagon, in time $\mathcal{O}(n^2)$.
- If a set P of n points has no empty pentagon, one can construct $QG\ P$ in time $\mathcal{O}(n^2)$.











Main Result

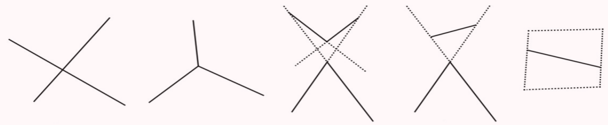
The triangulations of a finite point set from a flip graph that can be embedded isometrically into a hypercube, if and only if the point set has no empty convex pentagon. As a consequence, flip distance in such point sets can be computed efficiently.

Computing and estimating flip distance

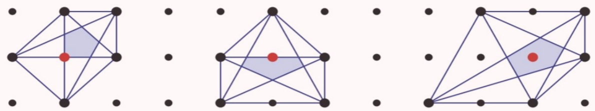
- Let T, T' be any triangulations of P
- Form a complete bipartite graph with the diagonals of T on one side and the diagonals of T' on the other
- Label the edge between any two diagonals by the distance between those diagonals in $QG\ P$
- Let M be the minimum weight of a perfect matching in this cimplete bipartite graph

M can be computet in polynomial time, and provides an underestimate of the true flip distance between T and T'

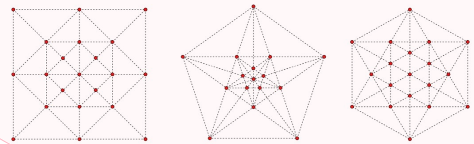
Examples



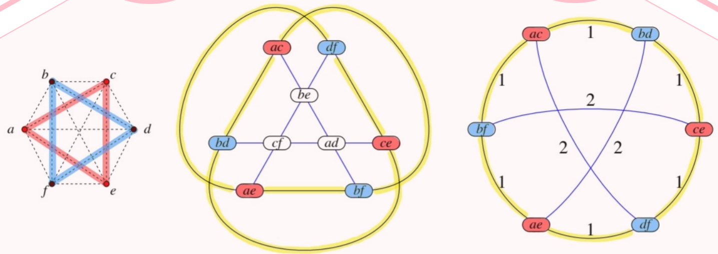
Continous point sets with no pentagons



Any lattice pentagons has another lattice point in it



Point sets with no empty pentagon derived from nested regular polygons



A hexagon (left), its $QG\ P$ (center) and the complete bipartite graph of distances in $QG\ P$ from $\{ac, ae, ce\}$ to $\{bd, bf, df\}$ (right)

Notice: For instance, if P has no empty pentagons, this estimate ends up calculating the flip distance exactly.