**10. Exercise sheet** **Introduction to Order Theory**
**Felsner/Micek** **Winter 2020/21**
Due for the exercise session: February 9., 2021

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(1) Try to find good time bounds for computing

a) The Ferrer's shape Fer($P$) of a poset $P$.

b) A maximum $k$-chain in $P$.

c) A maximum $\ell$ antichain in $P$.

(2) Think about analogs of the Greene-Kleitman theory for directed acyclic graphs.

(3) Given a 2-dimensional poset $P$, find good time bounds for computing

a) The skeleton of $P$

b) The Ferrer's shape Fer($P$) of $P$.

c) A maximum $k$-chain in $P$.

(4) This refers to lecture 28, so $P$ is 2-dimensional. Show that the $k$ chain $\mathcal{C}$ of $X$ obtained from a maximum $k-1$ chain $\mathcal{C}'$ of $S(X)$ is maximum without refering to $\ell$ antichains and orthogonal pairs.

Hint: Define a mapping from $k$ chains in $X$ to $k-1$ chains in $S(X)$.

(5) Visit the pages of Xavier Viennot (`www.viennot.org` and `www.xavierviennot.org`), stray around on these pages and tell us about findings you enjoyed.

(6) Find out about a result obtained by Logan–Shepp and Vershik–Kerov. What does it say about posets?

(7) Find an up-growing on-line width 2 poset $P$ such that the First-Fit algorithm (greedy) uses an unbounded number of chains.

(8) Within Lecture 29 we have seen that Algorithm has a strategy in on-line partition game on up-growing orders of width at most $w$ to use at most $\binom{w+1}{2}$ chains. Show that this is best possible (so devise a strategy for Presenter).

(9) Find the value of the on-line antichain partition game on interval orders of height at most $h$ presented with representation.

(10) Find the value of the on-line chain partition game on semi-orders of width at most $w$ (presented without representation)