5. Exercise sheet **Introduction to Order Theory**
Felsner/Micek **Winter 2020/21**

Due for the exercise session: December 8, 2020

---

(1) Consider disjointness graphs of axis-aligned rectangles in the plane. Prove that $\chi = \mathcal{O}(\omega \log \omega)$ for graphs in this class. *Hint*: divide & conquer.

(2) Consider a graph whose vertices are axis-aligned rectangles in the plane and two vertices are adjacent iff the corresponding rectangles cross. In the Asplund-Grnbaum we have seen that this crossing graph has a transitive orientation, i.e., we have defined a crossing poset on the rectangles. What is the maximum dimension of a crossing poset?

(3) Now show that the class of intersection graphs of unit-length segments in the plane is χ-bounded. You may use without proof that the class of intersection graphs of grounded segments is χ-bounded.

A thrackle is an embedding of a graph in the plane, such that each edge is a Jordan arc and every pair of edges meet exactly once. Edges may either meet at a common endpoint, or, if they have no endpoints in common, at a point in their interiors (in that case the common point must be a crossing).

(4) Find thrackle embeddings of:

   a. odd cycles;

   b. even cycles of length at least 6;

   c. trees.

(5) Show that a convex geometric graph (vertices are the corners of a convex polygon) with $n \geq 2k+1$ vertices which has no $k+1$ pairwise crossing edges can have $k(2n-2k-1)$ edges. (This bound is actually tight).

(6) If R is a circular sequence with entries from a set of $n > 1$ symbols such that no two adjacent entries are identical and R contains no circular subsequence of type $abab$, then the length of R is at most $2n-2$. (Such a sequence is called a circular Davenport-Schinzel sequence of order 2. Davenport-Schinzel sequences are a strong combinatorial tool to bound complexities in geometric settings),

(7) Two edges of a geometric graph are *parallel* if they are not crossing and their convex hull is a quadrangle. Show that a geometric graph without parallel edges can have at most $2n-2$ edges. *Hint*: Use the previous exercise.

(8) Show that given a family of n convex compact sets in the plane, one can always find $n^{1/5}$ of them which are either pairwise intersecting or pairwise disjoint. *Hint*: Think of segments first.