1. Übung "Graphen und Geometrie" **SoSe 2018**

Stefan Felsner Aufgaben für Do. 26. April

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(1) Determine the crossing number of the Petersen graph.

(2) Let $Z(m,n) = \lfloor \frac{m}{2} \rfloor \lfloor \frac{m-1}{2} \rfloor \lfloor \frac{n}{2} \rfloor \lfloor \frac{n-1}{2} \rfloor$ and show that $K_{m,n}$ has a drawing with $Z(m,n)$ crossings.

(3) Let Γ be a drawing of $K_{m,n}$ and let M, N be the bipartition. Assume that there are $u, u' \in M$ such that the graph induced by all edges incident to u and u' has no crossing. Prove the following inequality:

$$\mathsf{cr}(\Gamma) \geq \mathsf{cr}(K_{m-2,n}) + (m-2)\mathsf{cr}(K_{3,n}).$$

Show that with the assumption that $\mathsf{cr}(K_{m-2,n}) = Z(m-2,n)$ and $\mathsf{cr}(K_{3,n}) = Z(3,n)$ this implies that $\mathsf{cr}(\Gamma) \geq Z(m,n)$.

(4) Show that $\mathsf{cr}(K_{3,n}) = Z(3,n)$ for all n.

(5) Prove the following inequality for all $n \geq r > 3$:

$$\mathsf{cr}(K_n) \ \geq \frac{n(n-1)(n-2)(n-3)}{r(r-1)(r-2)(r-3)}\, \mathsf{cr}(K_r).$$

(6) Show that $\mathsf{cr}(K_6) = 3$, $cr(K_7) = 9$, and $cr(K_8) = 18$.
(The middle one is the most demanding, assume it when discussing K_8).

(7) Let $\mathsf{ip}(G) = |\{\{e,f\}\colon e \text{ and } f \text{ are independent edges in } G\}|$ and show that

$$\mathsf{cr}(G) \leq (1/6)\mathsf{ip}(G).$$