
9. Übung “Graphen und Geometrie”

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(1) Show that $\text{cr}(K_6) = 3$ as well as $8 \leq \overline{\text{cr}}(K_7) \leq 9$.

(2) Prove the following inequality for all $n \geq r > 3$:

$$\text{cr}(K_n) \geq \frac{n(n-1)(n-2)(n-3)}{r(r-1)(r-2)(r-3)} \text{cr}(K_r).$$

(3) Show that $\text{cr}(G) \geq 2m - 7n$.

Hint: Use the bound for the number of edges in 1-restricted drawings.

Use the above inequality to derive the crossing lemma with a better constant.

(4) Prove the following inequalities:

$$4 \binom{n+m-4}{n-2} \text{cr}(K_{n+m}) \geq \binom{n+m}{n} \text{cr}(K_{n,m}).$$

$$(m-2) \text{cr}(K_{n,m}) \geq m \text{cr}(K_{n,m-1}).$$

(5) Show that the limit of $\frac{\text{cr}(K_{n,n})}{n^4}$ exists.

(6) Determine $\text{cr}(K_{3,n})$ for all n .