Technische Universität Berlin Winter 2025/26
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Graph Theory (DS II) - Sheet 14

Exercise 14.1.
Even more triangle-free graphs with high chromatic number: Consider the Double Shift graphs $S(3,n)$. The vertices of $S(3,n)$ are triples (a,b,c) of integers $1 \leq a < b < c \leq n$ with edges $(a,b,c)(b,c,d)$. Show: $\chi(S(3,n)) \geq \log_2 \log_2 n$.

Exercise 14.2.
Given a family of intervals $(a_i, b_i) \subset \mathbb{R}$, $i = 1, \ldots, n$ on the number line. Define the graph with these intervals as the vertices and edges between overlapping intervals. Show that this graph is perfect. Such graphs are called *interval graphs*.

Exercise 14.3.
Let G be a perfect graph and let $L(G)$ be the line graph of G. Show that the complement of G and the complement of $L(G)$ are both perfect.

Exercise 14.4.
Let G be a chordal graph with $V = [n]$ such that $1, \ldots, n$ and $n, \ldots, 1$ are both perfect elimination orderings. Show that G is an interval graph. What other properties can you deduce about G?

Bonus Exercise
Place a set of n ants on the interval $[0,1]$ and give each ant a direction. The ants start walking with a speed of 1 in the given direction. If two ants collide, they both change direction and continue with speed 1. If an and reaches 0 or 1, it falls of the interval. What is the maximum amount of time an ant can be on the interval, depending on n?